- For this assignment, state which method of proof you are using for each problem.

1. (6 $\frac{\text{pts}}{}$ ea) Prove the following:

(a) The product of two rational numbers is a rational number.

(b) The average of two rational numbers is also rational.

2. (6 $\frac{\text{pts}}{}$) Prove that if x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

3. (6 $\frac{\text{pts}}{}$) Prove that if m and n are integers such $m + n$ is odd, then $3m + 5n$ is odd.

4. (6 $\frac{\text{pts}}{}$ ea) Prove or disprove each of the following. For each statement, assume a, b, and c are integers.

(a) If $a \mid (b + c)$, then $a \mid b$ and $a \mid c$

(b) If $a \mid b$ and $b \mid c$, then $a \mid c$

(c) If $a \mid b$ or $a \mid c$, then $a \mid bc$

(d) If $a \mid (bc)$, then $a \mid b$ or $a \mid c$

(e) If $ab \mid c$, then $a \mid c$ and $b \mid c$.

5. (6 $\frac{\text{pts}}{}$) Prove that if n and m are integers such that $n^2 + m^2$ is odd, then m is odd or n is odd.

6. (6 $\frac{\text{pts}}{}$) Prove that the sum of an irrational number and a rational number is irrational using a proof by contradiction.

7. (6 $\frac{\text{pts}}{}$) Prove or disprove: If x and y are irrational numbers, then xy is an irrational number.

8. (6 $\frac{\text{pts}}{}$) Prove: If n is an integer such that $n^2 + 7$ is even, then n must be odd.