

1. (10 pts) Let p , q , r , and t be the following following propositions:

p : “it is raining”

q : “the sun is shining”

r : “there are clouds in the sky”

t : “I am carrying an umbrella”

Translate the following English sentences into logical notation using p, q, r, t and logical operators ($\neg, \vee, \wedge, \rightarrow, \leftrightarrow$).

- (a) It is raining and I am carrying an umbrella.
 - (b) The sun is shining or there are clouds in the sky.
 - (c) I am carrying an umbrella if and only if it is raining.
 - (d) If it is raining, then the sun is not shining and there are clouds in the sky.
 - (e) If there are clouds in the sky or it is raining, then the sun is not shining and I am carrying an umbrella.
2. (9 pts) Let p , q , r , and t be the propositions in the previous problem. Translate the following into English sentences.

(a) $p \wedge r \wedge t$

(b) $(p \vee r \vee \neg q) \rightarrow t$

(c) $t \leftrightarrow (q \wedge \neg r)$

3. (6 pts) Give the converse of each of the following propositions.

(a) $r \rightarrow p$

(b) If I like apples, then I like apple pie.

(c) If $x + y < 0$, then $x < 0$ or $y < 0$

4. (6 pts) Give the contrapositive of each of the propositions in the previous problem.

5. (6 pts) Give the inverse of each of the propositions in problem 3.

6. (8 pts) Determine the truth values of the following statements:

- (a) If $2+3=4$ then $3+5=8$.
- (b) If $2+3=4$ then $3+5=7$.
- (c) If $2+3=5$ then $3+5=8$.
- (d) If $2+3=5$ then $3+5=7$.

7. (10 pts) Construct truth tables for the following.

- (a) $(\neg r) \rightarrow (p \vee q)$
- (b) $\neg(r \rightarrow (p \wedge q))$

8. (12 pts) Disprove the following assertions by providing a counterexample.

- (a) $2^n - 1$ is prime for every $n \geq 2$
- (b) $2^n + 3^n$ is prime for all natural numbers n
- (c) If p is prime, then $2p + 1$ is prime
- (d) $\forall n \geq 2 (n^2 > 2^n)$

9. (12 pts) [1.6.3 from zyBooks] Consider the following statements in English. Write a logical expression with the same meaning. The domain of discourse is the set of all real numbers.

- (a) There is a number whose cube is equal to 2.
- (b) The square of every number is at least 0.
- (c) There is a number that is equal to its square.
- (d) Every number is less than or equal to its square.

10. (12 pts) [1.6.4 from zyBooks] The domain for this problem is the elements a, b, c, d . The table below shows the value of three predicates for each of the elements in the domain. For example, $Q(b)$ is false because the truth value in row b , column Q is **F**.

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Which statements are true? Justify your answer (likely with an example or counterexample).

- (a) $\forall x P(x)$
 - (b) $\exists x P(x)$
 - (c) $\forall x Q(x)$
 - (d) $\exists x Q(x)$
 - (e) $\forall x R(x)$
 - (f) $\exists x R(x)$
11. (10 pts) [1.7.3 from zyBooks] In the following question, the domain of discourse is the set of employees at a company. One of the employees is named Sam. Define the following predicates:

$T(x)$: x is a member of the executive team

$B(x)$: x received a large bonus

Translate the following English statements into a logical expression with the same meaning.

- (a) Someone did not get a large bonus.
- (b) Everyone got a large bonus.
- (c) Sam did not get a large bonus even though he is a member of the executive team.
- (d) Someone who is not on the executive team received a large bonus.
- (e) Every executive team member got a large bonus.