

All of the items below are examples of what tries to get passed off as a solution to following:

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

Only one is an honest-to-goodness, well-written, correct solution. The others are common attempts at solutions, which fall short. If you haven't done so prior to this point, it is time to note what a solution is, and how to write one.

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

$$\begin{aligned}\frac{f(x+3) - f(x)}{3} &= \frac{2(x+3)^2 + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2(x^2 + 6x + 9) + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2x^2 + 12x + 18 + 5x + 15 - 2x^2 - 5x}{3} \\ &= \frac{12x + 33}{3} \\ &= \boxed{4x + 11}\end{aligned}$$

This is an actual solution which is easy to follow and makes mathematical sense. It begins with a correct statement, which is two expressions which are in fact equal to each other, separated by a symbol (“=”) which designates that. It then proceeds down the page, not to the right, with each subsequent line beginning with an “=” below the same symbol on the previous line. This is legitimate and acceptable because until something new is written on the left-hand side, there is the convention that we are still referring to the last thing written on that side. So, the last line of this solution is actually taken to be $\frac{f(x+3)-f(x)}{3} = 4x + 11$, or to be clear, “f of x plus 3, minus f of x, divided by three is equal to 4 x plus 11.” Notice how, when read aloud, it is a complete sentence!

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

$$\begin{aligned} &= \frac{2(x+3)^2 + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2(x^2 + 6x + 9) + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2x^2 + 12x + 18 + 5x + 15 - 2x^2 - 5x}{3} \\ &= \frac{12x + 33}{3} \\ &= \boxed{4x + 11} \end{aligned}$$

This solution is incomplete. What are you computing? What is equal to these lines of things? Are they equal to each other? All equal to something else? *What do you mean?* Starting work with “=” is nonsense. An equal sign needs to be between two expressions which **are equal**, certainly not between one expression and an empty void of space. This reads as an incomplete sentence. The last line reads “equals 4 x plus 11.” Okay, *what* is equal to $4x + 11$?

If the first solution is equivalent to “Barbara went to the grocery store.” then this one is equivalent to “went to the grocery store.” If someone simply said “went to the grocery store”, wouldn’t you ask “Who went to the store?”?

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

$$\begin{aligned} &\frac{2(x+3)^2 + 5(x+3) - (2x^2 + 5x)}{3} \\ &\frac{2(x^2 + 6x + 9) + 5(x+3) - (2x^2 + 5x)}{3} \\ &\frac{2x^2 + 12x + 18 + 5x + 15 - 2x^2 - 5x}{3} \\ &\frac{12x + 33}{3} \\ &\boxed{4x + 11} \end{aligned}$$

This solution is also incomplete. There is nothing to suggest any connection between these statements. Are they somehow related? Are they just a list of statements that may or may not have anything to do with one another? At the end, this supposed solution reads “ $4x + 11$ ”... that’s it, it just says “four x plus eleven.” This is far from a complete statement which provides an answer to the problem posed.

If the first solution is “Barbara went to the grocery store.”, then this one is just “store.”

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

$$\begin{aligned}\frac{f(x+3)-f(x)}{3} &= \frac{2(x+3)^2+5(x+3)-(2x^2+5x)}{3} = \frac{2(x^2+6x+9)+5(x+3)-(2x^2+5x)}{3} \\ &\Rightarrow \frac{2x^2+12x+18+5x+15-2x^2-5x}{3} = \frac{12x+33}{3} \\ &\Rightarrow \boxed{4x + 11}\end{aligned}$$

This one is hard to follow, misuses symbols (\Rightarrow), and simply doesn't flow in a coherent manner. It concludes with " $= 4x + 11$ ", and it is unclear what should be done with it. Is "equals four x plus eleven" supposed to follow from something else in the jumble of expressions before it?

This solution is sort of like having all the words to "Barbara went to the grocery store.", but not having them in a coherent order that would actually make a sentence. It would be like Barbara went to
; the grocery
; store.

Note that, sure the intended meaning can be gleaned from this collection of words and extraneous, misused symbols, but it it hardly a sentence.

Simplify the difference quotient $\frac{f(x+3) - f(x)}{3}$ for the function $f(x) = 2x^2 + 5x$.

$$\begin{aligned}\frac{f(x+3) - f(x)}{3} &= \frac{2(x+3)^2 + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2(x^2 + 6x + 9) + 5(x+3) - (2x^2 + 5x)}{3} \\ &= \frac{2x^2 + 12x + 18 + 5x + 15 - 2x^2 - 5x}{3} \\ &= \frac{12x + 33}{3} \\ &= \boxed{4x + 11}\end{aligned}$$

This supposed solution basically has scratch work in the middle of it. The second, third, and fourth lines are related to what is above and below them, but again, are not written in a correct, coherent way. The denominator is simply dropped for a couple lines, only to reappear when convenient, as does the equal sign, making this a mess. Having this work in the middle which is related, but not equivalent to what surrounds it, is like putting Barbara's shopping list in the middle of the sentence saying that she is going to the store— "Barbara went milk, eggs, potatoes to the grocery store."