1. What is the largest rectangular area that 80 feet of fencing can enclose?

2. What is the minimum amount of fencing that can be used to enclose a 400 square foot rectangular area?

3. A rectangle has one side on the x-axis and two vertices on the curve $y = \sqrt{1 - x^2}$. What is the maximum area that such a rectangle can have? The minimum area?

4. What is the maximum area a right triangle with a hypotenuse of length L can have?

5. A landscape architect plans to enclose a 5000 square yard rectangular region in a botanical garden. She will use shrubs costing \$25 per yard along one side and fencing costing \$10 per yard along the remaining three sides. Find the dimensions of the enclosure which minimize the total cost of the construction. [Round your answers to the nearest 0.1 yard]

6. Find two numbers x and y whose difference is D and whose product is minimum.

7. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What is the largest area that can be enclosed?

8. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is a fixed 384 cm^2 find the dimensions of the poster with the smallest total area.

9. Bill wishes to construct a storage shed which has a square base, and a total volume of 1200 ft^3. The cost per square foot of the materials is \$5 for the floor, \$3 for the sides, and \$2 for the roof. What is the minimum cost of constructing the shed?

10. A commercial cattle company currently allows 20 steer per acre of grazing land. On average a steer weighs 2000 pounds at the market. Estimates by the Department of Agriculture indicate that the average weight per steer will be reduced by 50 pounds for each additional steer added per acre of grazing land. How many steer per acre should be allowed in order to optimize the total market weight of the cattle?

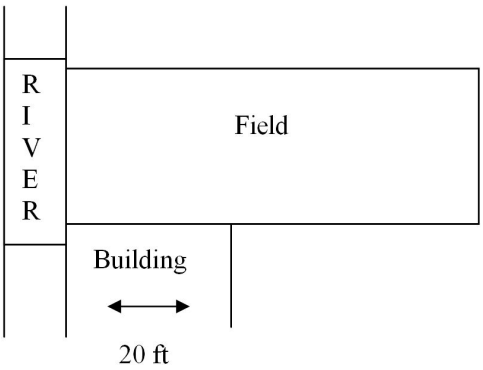
11. A company manufactures right circular cylindrical barrels. The top and bottom of the barrels are to be made with material that costs \$10 per square foot and the rest is made with material that costs \$8 per square foot. If each barrel is to hold 5 cubic feet, find the dimensions of the barrel that will minimize the total cost.

12. A window consisting of a rectangular region topped by a semicircle is to have a perimeter of 10 feet. Find the radius of the semicircle if the area of the window is to be a maximum.

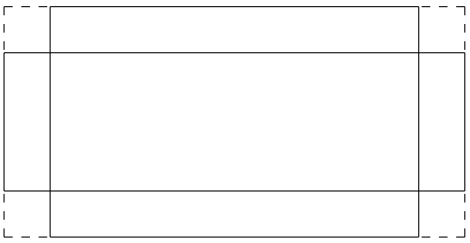
13. Bob wishes to construct a storage shed which will have a total volume of 1200 ft^3. The shed is to be designed so that the length will be twice the width. The cost per square foot of the materials is \$5 for the sides, and \$2 for the roof (no flooring is needed; dirt will be just fine). What is the minimum cost of constructing the shed?

14. Why might Bob (see previous problem) decide to spend a little more money to construct his shed than the actual minimum cost?

15. A rectangular field as shown below is to be bounded by a fence. Find the dimensions of the field with maximum are that can be enclosed with 1000 feet of fencing.



16. An open-top storage container is going to be constructed by cutting the corners out of a 4 ft $\times$ 8 ft sheet of plywood, and "folding" the edges up to form the sides of the container. What is the maximum volume of the resulting container? How much plywood is unused in making the container with the maximum volume?



17. The operating cost of a truck is $12 + \frac{x}{6}$ cents per mile when the truck travels at a speed of x miles per hour. Due to road construction, the truck can only travel between 30 and 60 miles per hour. If the truck driver earns \$8 per hour, what is the most economical speed to operate the truck on a 400 mile trip? What is the minimum cost?

18. With no concern for speed limits, what is the most economical speed? What is the minimum cost?

Answers

1. 400 ft^2

2. 80 ft

3. maximum: 1 unit^2
   minimum: 0 unit^2

4. $\frac{L^2}{4}$ unit^2

5. 53.5 yd $\times$ 93.5 yd

6. $\frac{D}{2}$ and $-\frac{D}{2}$

7. 720,000 ft^2

8. 24 cm $\times$ 36 cm

9. \$2139.81

10. 30 steer per acre

11. radius $\approx$ .86 ft
    height $\approx$ 2.15 ft

12. $r = \dfrac{10}{4+\pi} \approx 1.4$ ft

13. \$2060.49

14. The dimensions which minimize the cost of Bob's shed are impractical– It would have a base approximately 13.1 ft $\times$ 26.2 ft, but would only be about 3.5 feet tall!

15. 255 ft $\times$ 510 ft

16. The maximum volume is approximately 12.3 ft^3.
    There is approximately 2.9 ft^2 of unused plywood.

17. 60 mph
    \$141.33

18. 69.3 mph
    \$140.37