1. Determine $\frac{d}{dx}\left(8\sqrt{x^3}+6x-12\sqrt[3]{x}\right)$

2. What is the instantaneous rate of change of the function $f(x)=x^3-6x+\sqrt{x}$ when $x=4$?

3. Let $g(x)=x^3\left(2x^4+5x-6\right)$. Find $g'(x)$.

4. Determine $\frac{dy}{dx}$ for $y=\left(x^2-3x\right)\left(x^5+4x+12\right)$

5. What is the slope of the line which is tangent to the graph of $f(x)=2x^3-6x^2-5x+7$ when $x=-2$?

6. What is the slope of the curve given by $y=\frac{3}{2}x^{4/3}-16\sqrt{x}-\frac{1}{2}x^2+5x$ at the point $(64,-1472)$?

7. Determine the equation of the tangent line to the graph of $f(x)=x^2+5$ when $x=3$

8. Determine for $\dfrac{d}{dx}\left(\dfrac{6x^7+5x^3+7}{3x^2}\right)$

9. Determine the derivative of each of the following.

   (a) $f(t)=9t^4-8t^2+6t$

   (b) $z=8y^2-5y$

   (c) $C(q)=6\sqrt{q}+\frac{100}{q}$

   (d) $P(q)=-.1q^2+8.1q-7\sqrt{q}+4$

10. Determine the derivative of the following functions.

    (a) $f(x)=\dfrac{9x-5}{6-4x}$

    (b) $g(x)=\dfrac{x^2+4}{8x+3}$

    (c) $y=\dfrac{3x+2}{\sqrt{x}}$

    (d) $f(x)=\dfrac{x^2-3x+4}{x}$

    (e) $g(x)=\dfrac{15x^4+5x^3+9x^2+24x+17}{3}$

    (f) $h(x)=x^{\sqrt{2}}-4x^{\pi}+\frac{1}{2}x^{4.814}$

11. Determine $f'(0)$ for $f(x)=\left(x^4+5x^3-7x^2+11x-8\right)\left(2x^2-5x+7\right)$

12. Find $f'(1)$ for $f(x)=\left(x^2+5x\right)(8x-6)\left(x^3-7x^2-4x+9\right)$

13. Determine the slope of the tangent line to the graph of $y=\dfrac{5x^2+3}{2x^2-1}$ at $x=3$.

14. Find the equation of the line which is tangent to the graph of $y=\dfrac{9x}{8-5x}$ when $x=4$. Write your answer in slope-intercept form.

15. Determine the derivative of $f(x)=\dfrac{\left(x^2+2\right)(3x+4)}{2x-1}$

16. Determine $f'(x)$ if $f(x)=x(\sin x)(\cos x)(\tan x)(\sec x)(\csc x)(\cot x)$

17. Find $r'(x)$ for $r(x) = \ln\left(x\cos^2\left(e^{4x}+5\sin x\right)+x\sin^2\left(e^{4x}+5\sin x\right)\right)$

18. Use the table of values to compute the following.

x	-1	0	1	2	3	4
$f(x)$	5	2	$1/3$	-1	4	8
$f'(x)$	3	4	$2/3$	$1/2$	6	5
$g(x)$	2	-1	3	5	8	1
$g'(x)$	-4	6	4	3	$1/2$	7
$h(x)$	6	-5	1	4	5	2
$h'(x)$	-7	3	-2	6	$1/3$	3

(a) $k'(1)$ for $k(x) = x^2 f(x)$

(b) $m'(2)$ for $m(x) = \dfrac{f(x)g(x)}{x}$

(c) $w'(4)$ for $w(x) = \dfrac{\sqrt{x}}{h(x)}$

(d) $p'(-1)$ for $p(x) = \dfrac{g(x)}{xf(x)}$

(e) $d'(0)$ for $d(x) = g(x)f(x)h(x)$

(f) $q'(4)$ for $q(x) = f(x)\sqrt{x}$

Determine the derivative of the following functions.

19. $f(t) = \dfrac{t}{\sqrt{t^3+1}}$

20. $f(x) = \dfrac{x^2+1}{x^3}$

21. $z = (x+1)^3(5-x)^4$

22. $g(x) = \left(1+\frac{1}{x}\right)^{\sqrt{7}}$

23. $z = \log(10^{2m})$

24. $g(x) = \dfrac{xe^{5x}}{e^{3x}}$

25. $f(x) = \dfrac{5x^2}{(2-x)^3}$

26. $h(x) = e^{\sqrt[3]{6x+5}}$

27. $q(x) = e^{(7-3x)^2}$

28. $y = 3^{x^2}$

29. $p(x) = \ln\left(x^2\right)$

30. $f(x) = \left(e^x\right)^2$

31. $w(x) = \left(\ln(x)\right)^2$

32. $k(x) = e^{x^2}$

33. $g(x) = \ln\left(\ln(x)\right)$

34. $m(x) = (\ln(6))^x$

35. $x(r) = \sqrt{3r} + 3\sqrt{r} - \sqrt{\frac{3}{r}} + \sqrt{3}$

36. $h(x) = \dfrac{x^3 - 5x + 2}{\sqrt{2x+1}}$

37. $f(t) = \ln\left(\sqrt{t^2+1}\right)$

38. $f(x) = \dfrac{x^2 - 6x}{(x+1)^{-1}}$

39. $g(x) = \ln\left(3x^2\right)$

40. $b(x) = \dfrac{x^5 + 5x^4 + 2x^2}{x^2}$

41. $f(x) = \log_5\left(x^2 + 3x\right)$

42. $y = x^2 \ln(x^3)$

43. $f(x) = \dfrac{\ln(x)}{x^3 e^{4x}}$

44. $f(x) = \begin{cases} \sin x & x < 0 \\ x & 0 \le x < 4 \\ \frac{x^2}{8} & x \ge 4 \end{cases}$

45. $y = \sin\left(e^x\right)$

46. $f(x) = \sin(\sin(x))$

47. $T(t) = \sqrt{\sin\left(\sqrt{t}\right)}$

48. $y = \dfrac{\sin x}{1 + \sin^2 x}$

49. $W(u) = u^3 \sin(nu)$

50. $f'(1)$ if $f(x) = \tan^{-1}(2x)$

51. $g(x) = \sin\left(\frac{6\pi}{7}\right)$

52. $w(x) = \cos(\sin^{-1}(x))$

53. $f'(-3)$ if $f(x) = \ln x$

54. $\dfrac{d}{dt}\left(\dfrac{14e^x - 3x^8}{\sin^{-1}(x)}\right)$

55. $m(x) = \cos\left(e^{1-x}\right)$

56. $g(y) = \ln\left(\cos\left(y^2\right)\right)$

57. if $f(\theta) = \dfrac{\cos^2\theta}{1 - \sin^2\theta}$

58. $h(x) = \cos^{-1}(x^2)$

59. $k(x) = \left(\cos^{-1} x\right)^2$

60. Use the values in the table to answer the questions below.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$	$f''(x)$
0	0	1	2	-1	4	-5	0
1	3	2	1	3	-2	-4	-4
2	1	0	3	-2	3	2	1
3	2	3	0	4	2	-3	2

(a) Determine the slope of the line tangent to $y = f(x)g(x)$ at $x = 1$.

(b) Determine whether $y = h\left(g(x)\right)$ is increasing or decreasing at $x = 3$.

(c) Find the equation of the tangent line to $y = f\left(g(x)\right)$ at $x = 2$.

(d) Find $u'(1)$ if $u(x) = \sqrt{h(x) + 3}$.

(e) Determine $q'(2)$ for $q(x) = h^{-1}(x)$.

(f) Determine whether $y = \left(f(x)\right)^2$ is concave up or down at $x = 1$.

(g) Find the slope of $y = \dfrac{g(x)}{x^3}$ at $x = 2$.

(h) Find $m'(4)$ for $m(x) = h\left(\sqrt{x}\right)$.

(i) Find the slope of the tangent line to $y = e^{g(x)}$ at $x = 0$.

(j) Find $k'(1)$ for $k(x) = h(\ln x)$.

(k) Let $y = \dfrac{f(x)h(x)}{xg(x)}$. Compute $\left.\dfrac{dy}{dx}\right|_{x=3}$

Answers

1. $\frac{d}{dx}\left(8\sqrt{x^3} + 6x - 12\sqrt[3]{x}\right) = \left(12\sqrt{x} + 6 - \frac{4}{\sqrt[3]{x^2}}\right)$

2. $f'(4) = 42.25$

3. $g'(x) = 14x^6 + 20x^3 - 18x^2$

4. $\frac{dy}{dx} = 7x^6 - 18x^5 + 12x^2 - 36$

5. The slope is $f'(-2) = 43$

6. $\left.\dfrac{dy}{dx}\right|_{x=64} = -52$

7. $y = 6x - 4$

8. $\dfrac{d}{dx}\left(\dfrac{6x^7 + 5x^3 + 7}{3x^2}\right) = 10x^4 + \dfrac{5}{3} - \dfrac{14}{3x^3}$

9a. $f'(t) = 36t^3 - 16t + 6$

9b. $\frac{dz}{dy} = 16y - 5$

9c. $C'(q) = \frac{3}{\sqrt{q}} - \frac{100}{q^2}$

9d. $P'(q) = -.2q + 8.1 - \frac{7}{2\sqrt{q}}$

10a. $f'(x) = \dfrac{34}{(6 - 4x)^2}$

10b. $g'(x) = \dfrac{8x^2 + 6x - 32}{8x + 3}$

10c. $\dfrac{dy}{dx} = \dfrac{3x - 2}{2x^{3/2}}$

10d. $f'(x) = 1 - \frac{4}{x^2}$

10e. $g'(x) = 20x^3 + 5x^2 + 6x + 8$

10f. $h'(x) = \sqrt{2}x^{\sqrt{2}-1} - 4\pi x^{\pi-1} + 2.407x^{3.814}$

11. $f'(0) = 117$

12. $f'(1) = -242$

13. $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{66}{289}$

14. $y = \frac{1}{2}x - 5$

15. $f'(x) = \frac{[2x(3x+4) + 3(x^2+2)](2x-1) - 2(x^2+2)(3x+4)}{(2x-1)^2}$

$\cdots$

$f'(x) = \frac{12x^3 - x^2 - 8x - 22}{(2x-1)^2}$

16. $f'(x) = 1$

17. $r'(x) = \frac{1}{x}$

18a. $k'(1) = \frac{4}{3}$

18b. $m'(2) = 1$

18c. $w'(4) = \frac{-11}{8}$

18d. $p'(-1) = \frac{16}{9}$

18e. $d'(0) = -46$

18f. $q'(4) = 12$

19. $f'(t) = \frac{2 - t^3}{2\left(t^3+1\right)^{3/2}}$

20. $f'(x) = -\frac{1}{x^2} - \frac{3}{x^4}$

$= -\frac{x^2+3}{x^4}$

21. $\frac{dz}{dx} = (x+1)^2(5-x)^3(11-7x)$

22. $g'(x) = -\frac{\sqrt{7}}{x^2}\left(1 + \frac{1}{x}\right)^{\sqrt{7}-1}$

23. $\frac{dz}{dm} = 2$

24. $g'(x) = e^{2x} + 2xe^{2x}$

25. $f'(x) = \frac{20x + 5x^2}{(2-x)^4}$

26. $h'(x) = 2(6x+5)^{-2/3}e^{\sqrt[3]{6x+5}}$

27. $q'(x) = -6(7-3x)e^{(7-3x)^2}$

28. $\frac{dy}{dx} = 2x\ln(3)3^{x^2}$

29. $p'(x) = \frac{2}{x}$

30. $f'(x) = 2e^{2x}$

31. $w'(x) = \frac{2\ln(x)}{x}$

32. $k'(x) = 2xe^{x^2}$

33. $g'(x) = \frac{1}{x\ln(x)}$

34. $m'(x) = \ln\left(\ln(6)\right)\left(\ln(6)\right)^x$

35. $x'(r) = \frac{\sqrt{3}}{2\sqrt{r}} + \frac{3}{2\sqrt{r}} + \frac{\sqrt{3}}{2r^{3/2}}$

$= \frac{\sqrt{3}x + 3x + \sqrt{3}}{2x^{3/2}}$

36. $h'(x) = \frac{5x^3 + 3x^2 - 5x - 7}{(2x+1)^{3/2}}$

37. $f'(t) = \frac{t}{t^2+1}$

38. $f'(x) = 3x^2 - 10x - 6$

39. $g'(x) = \frac{2}{x}$

40. $b'(x) = 3x^2 + 10x$

41. $f'(x) = \dfrac{2x+3}{\ln 5\,(x^2+3x)}$

42. $\frac{dy}{dx} = 2x\ln(x^3) + 3x$

43. $f'(x) = \dfrac{1 - 3(\ln x) - 4x(\ln x)}{x^4 e^{4x}}$

44.
$$f(x) = \begin{cases} \cos x & x < 0 \\ 1 & 0 \le x < 4 \\ \frac{x}{4} & x > 4 \end{cases}$$
Note the derivative is *not* defined at $x = 4$.

45. $\frac{dy}{dx} = e^x \cos(e^x)$

46. $f'(x) = \cos(\sin(x))\cos x$

47. $T'(t) = \dfrac{\cos(\sqrt{t})}{4\sqrt{t\sin(\sqrt{t})}}$

48. $\dfrac{dy}{dx} = \dfrac{\cos^3 x}{(1+\sin^2 x)^2}$

49. $W'(u) = 3u^2\sin(nu) + nu^3\cos(nu)$

50. $f'(1) = \frac{2}{5}$

51. $g'(x) = 0$

52. $w'(x) = \dfrac{-x}{\sqrt{1-x^2}}$

53. $f'(-3)$ is undefined – $\ln x$ is only defined for $x > 0$.

54. $\dfrac{d}{dt}\left(\dfrac{14e^x - 3x^8}{\sin^{-1}(x)}\right) = 0$

55. $m'(x) = e^{1-x}\sin(e^{1-x})$

56. $g'(y) = -2y\tan(y^2)$

57. $f'(\theta) = 0$

58. $h'(x) = \dfrac{-2x}{\sqrt{1-x^4}}$

59. $k'(x) = \dfrac{-2\cos^{-1} x}{\sqrt{1-x^2}}$

60a. $\left.\dfrac{dy}{dx}\right|_{x=1} = 0$

60b. $\left.\dfrac{dy}{dx}\right|_{x=3} = -6$, so $y = h(g(x))$ is decreasing at $x = 3$.

60c. $y = -3x + 6$

60d. $u'(1) = -1$

60e. $q'(2) = \frac{-1}{5}$

60f. $\left.\dfrac{d^2y}{dx^2}\right|_{x=1} = -6$, so the graph is concave down at $x = 1$

60g. $\left.\dfrac{dy}{dx}\right|_{x=2} = \dfrac{3}{8}$

60h. $m'(4) = \frac{1}{2}$

60i. $\left.\dfrac{dy}{dx}\right|_{x=0} = 4e$

60j. $k'(1) = -5$

60k. $\left.\dfrac{dy}{dx}\right|_{x=3} = \dfrac{-2}{3}$

