

Linear Algebra - Math215-005
Supplemental HW-3 - Due 5/4/11

As always, in order to receive full credit, you must show *ALL* of your work!

1. For each of the following 12 statements, determine whether the statement is **TRUE** or **FALSE**. Indicate your answer by writing the entire word 'TRUE' or 'FALSE' at the start of your answer. In the space below each statement,
 - (a) If you marked the statement TRUE:
provide a brief justification, explanation, or proof as to why the statement is TRUE.
 - (b) If you marked the statement FALSE:
either provide a counter-example *or* a brief justification, explanation, or proof as to why the statement is FALSE.
Each problem is worth 2 points, with a bonus point for the proper formatting of your assignment.
 - i. For any $m \times n$ matrix A , its null space, $\text{Nul } A$ is a subspace of \mathbb{R}^m .
 - ii. For any $m \times n$ matrix A , its column space $\text{Col } A$ is the range of the linear transformation $x \mapsto Ax$ with domain \mathbb{R}^n .
 - iii. The set $\mathcal{B} = \{b_1, \dots, b_p\}$, a subset of a vector space V , is a basis for the subspace $H = \text{Span}\{b_1, \dots, b_p\}$ of V if and only if \mathcal{B} is a linearly independent set.
 - iv. A basis is a spanning set that is as large as possible.
 - v. Suppose \mathcal{B} , a set with n elements, is a basis for a vector space V . If x is in V , then the \mathcal{B} -coordinates of x is in \mathbb{R}^n .
 - vi. If V and W are 2 vector spaces with the same dimension, then there exists an isomorphism between them.
 - vii. A 5×7 matrix and a 7×5 matrix, both have *largest possible* rank equal to 7.
 - viii. The smallest possible dimension the null space of a 4×7 matrix can have is 3.
 - ix. If v_1 and v_2 are linearly independent eigenvectors of a matrix, then they correspond to distinct eigenvalues of that matrix.
 - x. If λ is an eigenvalue of an invertible matrix A , then $\lambda^{-1} = 1/\lambda$ is an eigenvalue of A^{-1} .
 - xi. If $(\lambda + 2)$ is a factor of the characteristic polynomial of a matrix A , then 2 is an eigenvalue of A .
 - xii. Suppose W is a subspace of \mathbb{R}^n . If $x \in W$ and $x \in W^\perp$, then x is the $n \times 1$ zero vector in \mathbb{R}^n .

2. For each of the statements that you marked **FALSE**, give a similar statement which is **TRUE** and provide a brief justification, explanation, or proof of why this new statement is true. There may be more than one similar true statement for each **FALSE** statement, you need only provide one. These can sometimes be found by modifying a number or letter (variable or parameter) in the previously **FALSE** statement, other times you may need to add another condition about one of the objects in a statement in order to make it true.
This will be worth 1 bonus point per false statement that is properly corrected and justified.

As a reminder, the same formatting guidelines as our previous supplemental homework assignments, must also be followed on this assignment. For completeness, they are included below.

The problems listed below are to be completed on separate sheets of paper and submitted during class on 5/4/11.

The following formatting guidelines **MUST** be strictly adhered to. Failure to do so will result in a penalty toward your grade...Submissions failing on more than one of the following guidelines may receive **NO** credit on the assignment.

Submission Guidelines:

1. Homework of more than one page must be **STAPLED**. One staple in the upper left hand corner will suffice. Crazy corner origami, paper clips, and binder clips, are **NOT** the same thing as a staple!
2. All assignments must be submitted on standard, full sized paper: 8.5" by 11".
3. Any **FRINGES** from tearing the paper out of a notebook, **MUST BE REMOVED!**
4. The upper right hand corner of the first page should have your **FIRST** and **LAST** names.
5. The upper left hand corner of the page should have the **COURSE** and **SECTION** numbers and the **DATE**.
6. You must write **NEATLY** and **LEGIBLY**. Be sure to always **SHOW YOUR WORK!**
7. Assignments must be completed one problem **BELOW** the previous one, not in multiple columns, with at least **ONE LINE** of **SPACE** in between.
8. If you use a pen that bleeds through the paper, only write on one side, don't write on both sides of the paper.