Math 129
Past Exam Questions

Here are some questions that appeared on common exams in past semesters. This is not a sample exam, but it is a reasonable guide to the style and level of common exam given by the U of A Mathematics Department.

1. a. Write the complex number $\sqrt{2} - i\sqrt{2}$ in the form $Re^{i\theta}$. Your answer must be exact.
b. Write the complex number $3e^{\frac{5\pi i}{3}}$ in the form $a + bi$. Your answer must be exact.

2. Simplify

$$e^{(3+4i)t}$$

to the form $a(t) + b(t)i$

3. Write $3 - 3i$ in the form $R\,e^{\theta i}$ with R and θ real numbers.

4. a) Write the complex number $2\sqrt{2} - i2\sqrt{6}$ in the form $Re^{\theta i}$ exactly.
b) Write the complex number $\pi e^{\frac{7\pi}{6}i}$ in the form $a + bi$ exactly.

5. Let α and β be the complex numbers

$$\alpha = 2 + \tfrac{\pi}{3}i, \text{ and } \beta = 1 + i.$$

Write each of the following complex numbers (exactly) in the form $a + b\,i$.
a) $\alpha\beta$ b) e^{α} c) β^{-1}

6. Write $e^{(3-i\frac{\pi}{3})\,t}$ in the form $a(t) + i\,b(t)$ where $a(t)$ and $b(t)$ are real.

7. a. Write the complex number $2 - i\sqrt{12}$ in the form $Re^{i\theta}$. Your answer must be exact.
b. Write the complex number $5e^{\frac{7\pi}{6}}$ in the form $a + bi$. Your answer must be exact.

8. a) Write the complex number $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ in the form $Re^{\theta i}$
b) Write the complex number $\left(2e^{-\frac{\pi}{4}i}\right)$ in the form $x + y\,i$

9. a) Write $4\,e^{3i}$ in the form $x + iy$. (Your answer need only be accurate to 3 decimal places)
b) Write $5 + 12\,i$ in the form $Re^{i\theta}$. (Your answer need only be accurate to 3 decimal places)

10. Give exact answers for both of the following: a. Write $1 + i$ in the form $Re^{i\theta}$.
b. Write $10e^{\frac{2\pi}{3}i}$ in the form $a + bi$.

11. Write $e^{2t-5t\,i}$ in the form $a(t) + i\,b(t)$ where $a(t)$ and $b(t)$ are real valued functions.

12. Write $-5 + 5\,i$ in the form $Re^{i\theta}$. Your answer must be exact

13. Evaluate: a) $\int \sin(3x)\,dx$ b) $\int \frac{1}{x^2-1}\,dx$ c) $\int \frac{2x}{\sqrt{1-x^2}}\,dx$
d) $\int e^{\frac{1}{2}x+1}\,dx$ e) $\int x(x+1)\;dx$

14 Find the exact value of a) $\int_0^1 x\,e^{x+1}\,dx$ and b) $\int_0^{\pi} \sin(\frac{x}{3})\sin(\frac{x}{3})\;dx$

15. Find $\int\;(1+\ln x)\sin^3(x\ln x)\;dx$ (Hint: Use the substitution $w = x\ln x$)

16. Find the Taylor polynomial of degree 6 that approximates the function
$f(x) = e^{x^2}$ near $x = 0$.

17. Evaluate
a) $\int x\,e^{x^2}\,dx$ b) $\int\;\frac{x+1}{x^2+1}\;dx$ c) $\int_{-1}^{1} x^4\,dx$
d) $\int\;\sin^2 x\;\;dx$ e) $\int\;\sqrt{x+1}\;dx$

18. According to a book of mathematics tables,

$$\int_0^{\pi} \ln(5 + 4\cos(x))\,dx = 2\pi\ln(2)$$

Use this formula and substitution $w = 4\arctan(x)$ to find an exact value of

$$\int_0^1 \frac{\ln(5 + 4\cos(4\arctan(x)))}{1+x^2}\,dx$$

19. The following result can be found in many mathematical tables

$$\int_0^{\infty} \frac{\cos(t)}{1+t^2}dt = \frac{\pi}{2e}.$$

Use substitution $t = \tan(s)$ and this formula to evaluate

$$\int_0^{\frac{\pi}{4}} \cos(\tan(2s))\,ds$$

20. Sketch a graph of the region between the graph of $y = e^{\frac{x}{2}} + e^{\frac{x}{2}}\sin x$ and the x-axis bounded by $x = 0$ and $x = \frac{3\pi}{2}$ and find its exact area.

21. Use the substitution $w = (x^4+3)^2$ to evaluate

$$\int x^3(x^4+3)^5\ e^{(x^4+3)^2}\ dx$$

You must show your work for full credit.

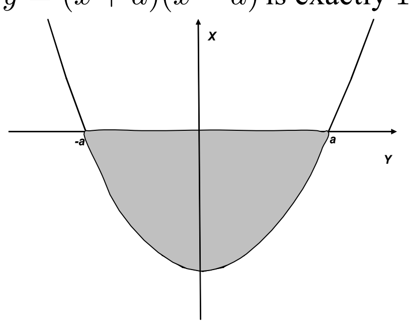
22. A biologist hypotheses that a certain fungus spreads by growing in a rough circle. The radius of the circle grows at a rate **inversely** proportional to the **area** of the circle. If the fungus starts in a circle with a radius 2 ft.. and has a radius of 2.2 ft one week later, find a formula for the radius of the circle after t weeks.

23. Let $a \geq 0$. Find

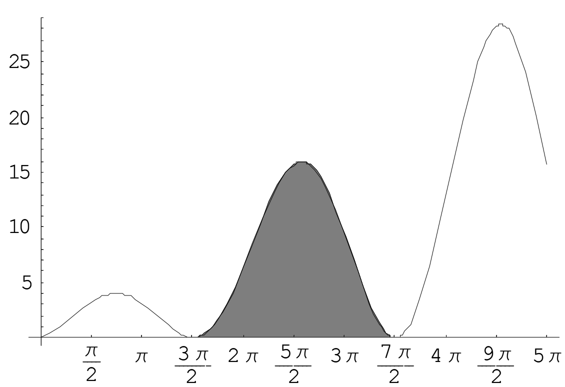
a) $\int \frac{dx}{1+a^2+2x+x^2}$ b) $\int \frac{dx}{1+a^2+2a+x^2}$

24. Find $\int \frac{\cos^3(\ln(2x))}{x}\ dx$ Hint: Start with the substitution $w = \ln(2x)$

25. Find a value for a so that the area between the x-axis and the curve $y = (x+a)(x-a)$ is exactly 18.



26. The graph of the function, $f(x) = x + x\ \sin(x)$ is given below. Find the exact area of the shaded region.



27. Sketch a graph of the curve $y = (x^2-16)\,e^x$. Find the exact area of the region below the x axis bounded by this curve. You must show your work.

28. Find the exact volume of the solid obtained by revolving one arc $(0 \leq x \leq \frac{1}{3})$ of the curve $y = \sin(3\pi x)$ about the x-axis. You must show your work to receive full credit.

29. Sketch the parabola $y = (x + \frac{\pi}{2})(x - \frac{\pi}{2})$ and the curve $y = \cos(x)$ showing their points of intersection. Find the area of the region between these two graphs.

30. Find the exact value of the volume of the solid obtained by revolving the region above the graph of $y = x^2$ and below the line $y = 9$ about the line $y = 9$. (You must show your work for credit.)

31. The following formula appears often in mathematics and its applications.

$$\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$$

Use this formula to evaluate

$$\int_m^\infty e^{-(\frac{t-m}{s})^2} dt$$

(Assume that m and s are non-zero constants.)

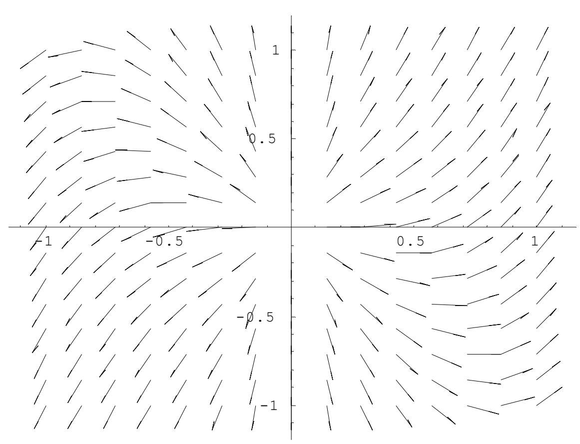
32. Find a function $y = y(x)$ so that $\frac{dy}{dx} = x(y^2 + 4)$ and $y(0) = 233$. Find the radius of convergence of the following power series: (Do not worry about the endpoints)

a) $\sum_{k=1}^{\infty} \frac{1}{k} x^k$ b) $\sum_{k=1}^{\infty} \frac{1}{2^k} x^k$

34. Find the **exact** volume of the solid obtained by revolving the region between the graph of $y = 4 - x^2$ and the x-axis about the line $y = 5$. Hint: use slices perpendicular to the x-axis.

35. Here is a slope field for the differential equation

$$\frac{dy}{dx} = F(x, y)$$



You **must** explain your answer for credit.
a) Could $y = x^2$ be a solution to the differential equation? **Yes** **No** Explain:
b) Could $y = x^3$ be a solution to the differential equation? **Yes** **No** Explain:
c) Could $y = -\sin(x)$ be a solution to the differential equation? **Yes** **No** Explain:

36. a) For what values of p does the improper integral $\int_1^\infty \frac{1}{t^p}\, dt$ converge?
b) For what values of p does the improper integral $\int_0^1 \frac{1}{t^p}\, dt$ converge?

37. For what value of a do you have

$$\int_0^\pi (x^2 + ax + 1)\cos(x)\, dx = 7\pi$$

You must show your work in a clear and logical manner.

38. Match the differential equation with one of its solutions. Hint: It cannot hurt to draw the slope fields and the graphs if you are stuck.

a) $\frac{dy}{dx} = \sqrt{|y^2 - 4|}$ $y = e^x + e^{-x}$
b) $\frac{dy}{dx} = \frac{y-5}{x^2y^2+1}$ $y = 5$
c) $\frac{dy}{dx} = y - x^3 + 6x$ $y = x^3 + 3x^2$
d) $\frac{dy}{dx} = y - \ln|y - x| + 1$ $y = e^x + x$

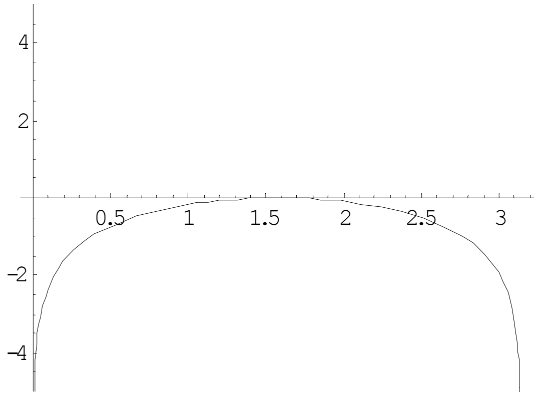
39. Circle the improper integrals that converge: (5 points off for each incorrectly marked integral up to a maximum of 20 points off.)

a) $\int_0^1 \frac{1}{x}\, dx$ b) $\int_1^\infty \frac{1}{x}\, dx$ c) $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$ d) $\int_0^1 \frac{1}{\sqrt{x}}\, dx$ e) $\int_1^\infty \frac{1}{x^2}\, dx$ f) $\int_0^1 \frac{1}{x^2}\, dx$

40. Mathematicians have found that the improper integral below converges and have determined its value

$$\int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx = -\frac{\pi}{2}\ln(2)$$

a) Explain why the integral in this formula is improper.
b) The graph of $f(x) = \ln(\sin x)$ between $x = 0$ and $x = \pi$ is



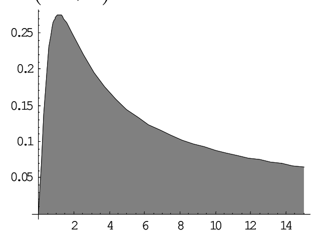
Shade in the area determined by this integral, and explain why the sign is negative in the formula.

c) Use substitution and the formula to evaluate

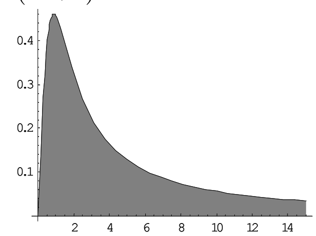
$$\int_0^{\pi} \ln(\sin \tfrac{t}{2})\, dt \;=$$

41. In each of the following, determine whether the area under the curve in the first quadrant is infinite or finite.

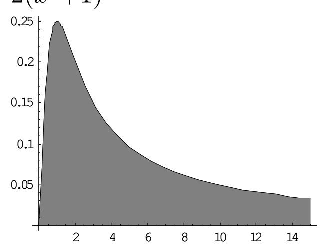
a) $y = \dfrac{x}{2(x^2+1)^{\frac{7}{8}}}$



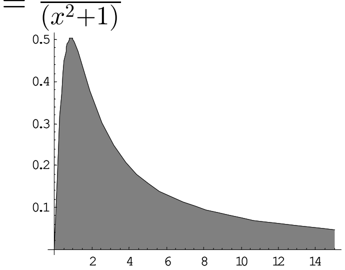
b) $y = \dfrac{x}{(x^2+1)^{\frac{9}{8}}}$



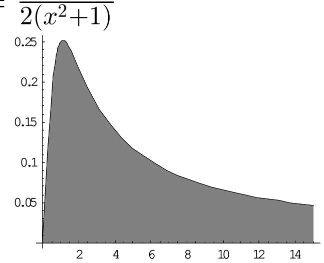
c) $y = \dfrac{x}{2(x^2+1)}$



d) $y = \dfrac{x^{\frac{7}{8}}}{(x^2+1)}$



e) $y = \dfrac{x^{\frac{9}{8}}}{2(x^2+1)}$



42. Do the following <u>improper</u> integrals converge or diverge? If one converges, give its value. You must justify your answer.

a) $\int_1^{\infty} \frac{dx}{(x+3)^3}$ b) $\int_1^{\infty} \frac{x^2+x}{x^3+x^2+1}\, dx$ c) $\int_2^{10} \frac{1}{(x-2)^2}\, dx$ d) $\int_2^{10} \frac{1}{\sqrt{x-2}}\, dx$

43. Find the exact area of the region between the graphs of the two functions

$$f(x) = \sin(\pi x) \text{ and } g(x) = x(x-1)$$

44. Find $\int \sin(ax)\, dx$ where a is a nonzero constant.

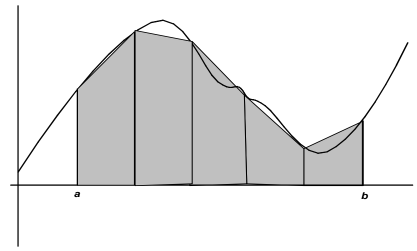
45. Find: $\int (\sin^6(x)\cos(x) + \cos^6(x)\sin(x))\, dx$

46. Evaluate the following integral exactly: $\int_0^2 x^3 e^{x^2}\, dx$

47. Find: $\int 4\ xe^{2x}\ dx$

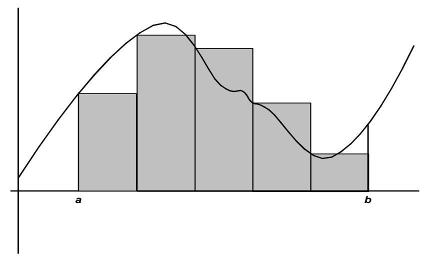
48. Match the graph with the approximation it illustrates:

A



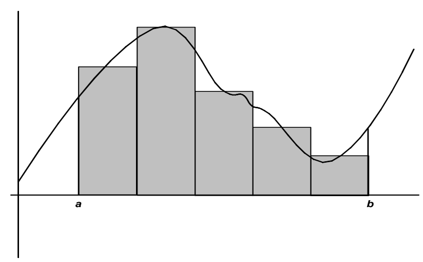
Right Hand Rule_______

B



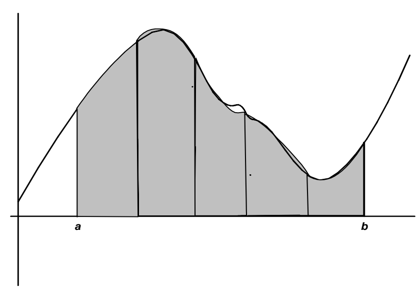
Left hand Rule_________

C



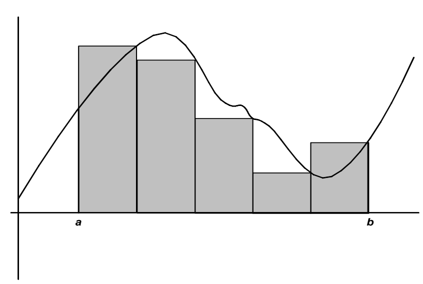
Midpoint Rule________

D



Trapezoid Rule_______

E



Simpson's Rule_______

49. Find a formula for the volume of the solid obtained by revolving one arc $(0 \leq x \leq \pi)$ of the curve $y = a\sin(ax)$ about the x-axis. Assume $a > 0$.

50. Does the following improper integral converge or diverge?

$$\int_3^4 \frac{d\theta}{(\theta-3)^3}$$

Give a reason for your answer; give a precise argument if you can.

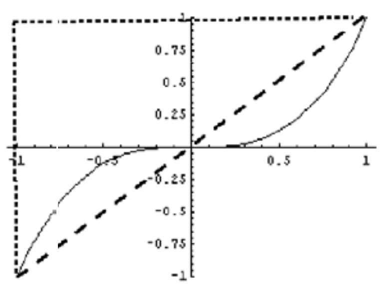
51. Does the following improper integral converge or diverge? You must give a complete justification of your answer. (Hint: You do not need to evaluate the integral to answer the question.)

$$\int_2^\infty \frac{d\theta}{\sqrt{\theta^3+1}}$$

52. If $y = f(x)$ is a function differentiable from $x = a$ to $x = b$, then the length of its graph from $x = a$ to $x = b$ (arc length) is given by

$$\text{ArcLength} = \int_a^b \sqrt{1 + (f'(x))^2}\ dx$$

Consider the curve $y = x^3$ between $x = 0$ and $x = 1$. Unfortunately the integral given by the formula above does not have a closed form, and so the arc length must be estimated numerically.



The straight line distance from $(-1, -1)$ to $(1, 1)$ is $2\sqrt{2}$ while the distance measured going straight up and then over is 4. Thus the arc length is between $2\sqrt{2}$ and 4.

a) Find a sharper upper bound and a sharper lower bound estimate of the arc length. (You must justify your answer completely for credit!)

b) Find an estimate of the arc length that is accurate to 3 places after the decimal.

53. You are given the following mathematical fact

$$\int_0^\infty \frac{x}{e^x+1}\, dx = \frac{\pi^2}{12}$$

Use this formula and substitution to find an exact value for

$$\int_0^\infty \frac{x}{e^{3x}+1}\, dx$$

54. Find $\int (x+1)\, e^{2x+2}\, dx$

55. Evaluate the following integral exactly: $\int_0^1 \frac{1+e^{3x}}{e^{3x}+3x}\,dx$.
56. The region bounded by $y = x^3$, $x = 2$, and $y = -1$ is revolved about the line, $y = -8$. Sketch a picture of the solid, and find its volume.

57. Sketch a picture of the solid obtained by revolving the region bounded by the curve $y = x^4$ and the line $y = x$ about the y-axis. Also find the volume of this solid. (Hint: you *may* want to use slices perpendicular to the $y-$ axis.)

58. Solve the following initial value problem for $s(t)$:
$\frac{ds}{dt} = \frac{s}{2t} + s\,t$ where $s(1) = e$

59. Does the following improper integral converge or diverge? If it converges, evaluate it exactly. If it diverges, explain why. (Approximate answers may earn partial credit, but only if accurate and completely justified.)

$$\int_2^\infty \frac{x^3}{x^4-1}\,dx$$

60. Does the following improper integral converge or diverge? If it converges, evaluate it exactly. If it diverges, explain why. (If it converges and you give an approximate answer, credit may be reduced if the approximation is not accurate and completely justified.)

$$\int_2^\infty \frac{x^3+1}{(x^4+4x+1)^2}\,dx$$

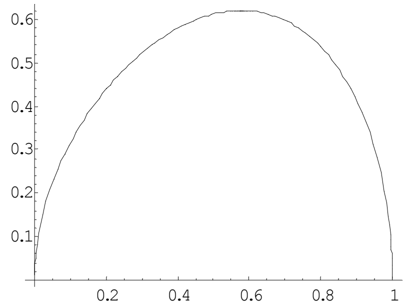
61. a) Evaluate the following integral exactly: $\int_1^4 x^3\, e^{x^2+1}\,dx$. (You must show work to earn credit. Poor approximations will **not** earn partial credit.)
b) Evaluate the following integral exactly: $\int \frac{1}{x^2+4x+8}\,dx$ (You must show work to earn credit.)

62. A cylindrical form is filled with a slow curing concrete. The base of form is 10 ft. in radius, and the height is 25 ft. While the concete hardens, gravity causes the density to vary from a density of 90 lbs/ft^3 at the bottom to a density of 50 lbs./ft^3 at the top. Assume that the density varies linearly from top to bottom, and compute the total weight of the resulting concrete column. (Hint: First write a linear function that gives the density at height h from the bottom.)

63 Dead leaves accumulate on the floor of a forest at a continuous rate of 4 grams per square centimeter per year. At the same time, these leaves decompose at the rate of 60% per year. Write a differential equation for the quantity of leaves (in grams per square centimeter) at time t. Use this differential equation to find the amount of dead leaves (in grams per square centimeter) which represents equilibrium in the system.

64. Solve the following initial value problem for $s(t)$:
$\frac{ds}{dt} = \frac{s}{2t} + s\,t$ where $s(1) = e$

65. The function $f(x) = \sqrt{x - x^3}$ has a graph that looks like



Approximate $\int_0^1 \sqrt{x - x^3}\ dx$ to two decimal digits of accuracy. Give an argument that shows that your answer has the required accuracy that includes an upper bound and a lower bound on the exact answer.

66. In this problem, you will approximate the value of the definite integral

$$\int_{-1}^{1} \frac{1}{x^6+6}\ dx.$$

a) Use 30 divisions to find an approximation using: Right hand sums, Left hand sums, The midpoint rule, The trapezoid rule
b) Use the above to give an upper and a lower bound on the exact value of the integral.
c) Justify your answer to part b.

67. Biologists have introduced a new variety of fish into a lake. They began by releasing 1000 fish. Their model predicts that the population will double in 18 months and then double again 27 months later (45 months after the start.)
Do you think that the biologists are using the exponential model

$$\frac{dA}{dt} = kA$$

or the logistic model

$$\frac{dA}{dt} = kA(C_0 - A)$$

for the population of the fish? Explain your answer

68. Find the Taylor polynomial of degree 2 that approximates the function $f(x) = \sqrt{x^3 + 1}$ near $x = 2$. **You must show your work for full credit.**69. A room with a southern exposure heats up during the morning. The temperature of the room increases linearly all morning so that it rises 1° F every 15 minutes. Early in the morning, a cup of coffee with a temperature of 180° F is placed in the room when the room temperature is 60° F. Newton's law of cooling states that the rate of change in the temperature of the coffee should be proportional to the difference in temperature between the coffee and the room.
a) Write a formula for the **temperature of the room** t minutes after the coffee placed there.
b) Write a differential equation that the **temperature of the coffee** satisifies.
c) Give specific initial conditions necessary to solve this problem.
You do not need to solve the differential equation.

70. Find the exact value of $\sum\limits_{k=0}^{\infty} \frac{5^k+1}{11^k}$. (Hint: Write the sum in terms of geometric series.)

71. a) Give an example of a convergent infinite geometric series. Explain why it converges, and say what it converges to.
b) Give an example of a divergent geometric series. Explain why it diverges.

72. Find the Taylor polynomial of degree 6 that approximates the function $f(x) = e^{x^2}$ near $x = 0$.

73. Recall that the arc length of the curve $y = f(x)$ from $(a, f(a))$ to $(b, f(b))$ is given by

$$\int_a^b \sqrt{1 + (f'(x))^2} \quad dx$$

Use this to **approximate** the length of the curve $y = \ln x$ from $(1, 0)$ to $(4, \ln 4)$. The approximation should include a strict lower bound and a strict upper bound, and it should be accurate to at least two decimal places. ***Explain how you determined your answer.***

74. As you should know, the Taylor Expansion of the sine function is

$$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \frac{1}{13!}x^{13} - \ldots\ldots.$$

Consider the function

$$g(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Find $g^{(10)}(0)$, the tenth derivative of $g(x)$ evaluated at $x = 0$. (You must show your work or explain your answer for credit.)

75.a) Find the Taylor expansion of $f(x) = \tan^{-1}(x)$ about $x = 0$ to **degree 3.**
b) Find the Taylor expansion of $f(x) = \tan^{-1}(x)$ about $x = 1$ to **degree 4.**

76. Consider the function given by

$$f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{k!}{(2k)!}(x-2)^k$$

a) What is $f(2)$? b) What is $f'(2)$? c) What is $f''(2)$?
d) What is the Taylor expansion of $f(2x)$ about $x = 1$?

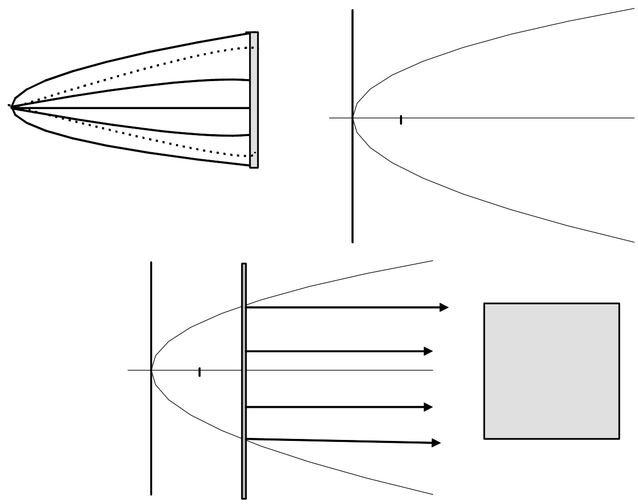
77. What is the radius of convergence of the power series $f(x) = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + 36x^6 + \ldots\ldots$

78. Find the Taylor expansion of $f(x) = (x+1)\sin(x)$ to degree 5 about the point $x = 0$.

79. Solve $\frac{dy}{dx} = e^y \sin(3x)$ for $y = f(x)$ when $f(\frac{\pi}{6}) = 0$.

80. a) Find the Taylor expansion of $f(x) = e^{2x}$ about $x = 0$ to degree 4.
b) Find the Taylor expansion of $f(x) = \sin^2 x$ about $x = 0$ to degree 6.

81. A solid is made so that its profile is a parabola and all its cross sections are squares. In other words, The base of the solid is the region bounded by a parabola and a line, and the cross sections perpendicular to the x-axis are squares with one side in the xy plane. We can draw this using the graph of the functions $f(x) = \sqrt{x}$ and $g(x) = -\sqrt{x}$.



If the solid extends from $x = 0$ to $x = 9$, what is the volume of the solid?

82. What functions have the Taylor expansions given below?

a) $\sum_{k=0}^{\infty} \frac{1}{k!} x^k$ b) $\sum_{k=0}^{\infty} x^k$ c) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$

d) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$ e) $\sum_{k=0}^{\infty} \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})\cdots(-\frac{2k-3}{2})}{k!} x^k$

83. Give the radius of convergence for the following power series:

a) $\sum_{k=0}^{\infty} \frac{1}{k+2} x^k$ b) $\sum_{k=0}^{\infty} (-1)^k \frac{1}{k!} x^k$ c) $\sum_{k=0}^{\infty} (-1)^k \frac{1}{k\, 2^k} x^k$

84. a)Does the series $\sum_{k=1}^{\infty} \frac{1}{k^3}$ converge? You must give a mathematically valid reason for your answer to receive any credit.
b) Does the series $\sum_{k=1}^{\infty} \frac{1+\cos(k)}{k^3}$ converge? You must give a mathematically valid reason for your answer to receive any credit.

85. Give the radius of convergence for the following power series: You must show your work.

a) $\sum_{k=0}^{\infty} \frac{1}{k+2}\, x^k$ b) $\sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)!}\, x^{2k+1}$ c) $\sum_{k=0}^{\infty} \frac{3^k}{2k}\, x^k$

86. Find Taylor polynomials of degree 6 that approximate the following functions near $x = 0$.

a) $f(x) = e^{x^2}$ b) $g(x) = \sqrt[3]{1+x}$ c) $h(x) = \dfrac{\cos x - 1}{x^2}$

87. Give the Taylor series expansion of the following functions about $x = 0$. Give as complete an answer as possible. For example, the Taylor expansion of $f(x) = \ln(x+1)$ should be written as either

$$f(x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \tfrac{1}{5}x^5 + \ldots + (-1)^{n+1}\tfrac{1}{n}x^n + \ldots$$

or

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1}\tfrac{1}{n}x^n$$

a) $f(x) = \frac{1}{1-x}$ b) $f(x) = e^x$ c) $f(x) = \sin x$ d) $f(x) = \cos x$

88. Find a function $f(x)$ so that

$$f(2) = f'(2) = f''(2) = f'''(2) = f''''(2) = f'''''(2) = 120.$$

89. Find the following Taylor expansions, and give a formula for the general term: You must explain how you obtained your answer.

a) $\sin(x)$ about $x = 0$

b) $\sin(\frac{\pi}{2} - x)$ about $x = 0$

c) $\sin(\frac{\pi}{2} - x)$ about $x = \frac{\pi}{2}$

90. Give the Taylor series expansion of the following functions about $x = 0$. Give as complete an answer as possible. For example, the Taylor expansion of $f(x) = \ln(x+1)$ should be written as either

$$f(x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \tfrac{1}{5}x^5 + \ldots + (-1)^{n+1}\tfrac{1}{n}x^n + \ldots$$

or

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1}\tfrac{1}{n}x^n$$

a) $f(x) = \frac{1}{1-x}$ b) $f(x) = \cos x$ c) $f(x) = \sin(\pi x)$ d) $f(x) = 3\, e^x$

91. a) Give the Taylor series expansion about $x = 0$ of the function $f(x) = \sin x$ Be sure to give an expression for the n-th term of the series.

b) What is the 5-th degree Taylor polynomial approximation of $f(x) = \sin x$ near $x = 0$?

92. Consider the initial value problem, $\frac{dy}{dx} = y \ln x$ where $y(e) = 1$.
Find a function $y = y(x)$ that satisfies these conditions.

93. Solve the following initial value problem:

$$\frac{dy}{dx} = y\sqrt{x+1} \quad \text{where} \quad y(8) = 5e^{18}$$

94. A metallic rod 20 cm in length is made from a mixture of several materials so that its density changes along its length. Suppose that the density of the rod at a point x cm from one end is

$$\delta(x) = 2 + 0.5\cos\left(\tfrac{\pi}{10}x\right) \text{ grams per cm of length}$$

a) Where is the rod the most dense? Where is it the least dense?
b) What is the total mass of the rod?

95. Simplify e^{1+2i} to the form $a + bi$. Give an exact answer.

96. Consider the function $\quad F(x) = \int_0^x e^{-t^2}\ dt$
a) What is $F'(x)$?
b) Give the first 5 nonzero terms of the power series expansion of the function $F(x)$ near $x = 0$.

97. A book of formulas states that for any p with $0 < p < 1$,

$$\int_0^\infty \frac{x^{p-1}}{1+x}\, dx = \frac{\pi}{\sin(p\pi)}$$

Use this formula and the substitution $x = au$ to obtain a formula for $\int_0^\infty \frac{x^{p-1}}{a+x}\, dx$

98. Suppose that at 12:00, noon, one summer afternoon, there is a power failure in your home in Tucson, and your cooling does not work. When the power goes out, it is 73° F in your house; the outside temperature is 108°. At 2:00 pm, the temperature in your house has climbed to 85°. Assume that the outside temperature is constant from 12:00 pm until 6:00 pm, and that your house obeys Newton's law of cooling. (Newton's law of cooling says that the rate of change of temperature of an object is directly proportional to the difference in temperature between the object and the ambient temperature.) Write a differential equation that the temperature of your home satisfies, and use it to predict the temperature of the house at 6:00 pm.

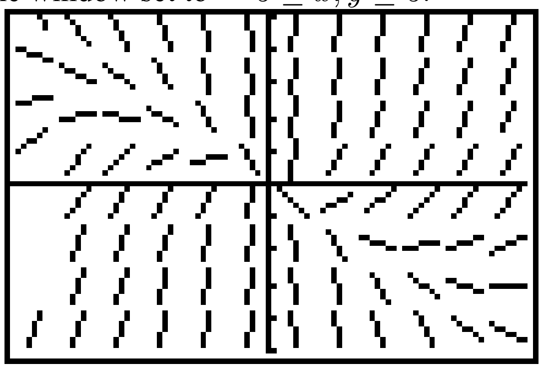
99. a) For what values of p does the improper integral $\int_1^\infty \frac{1}{t^p}\ dt$ converge?
b) For what values of p does the improper integral $\int_0^1 \frac{1}{t^p}\ dt$ converge?
c) For what values of p does the improper integral $\int_1^\infty \frac{1}{(t^2+1)^p}\ dt$ converge?

100. Solve the following initial value problem for $s(t)$: $\frac{ds}{dt} = \frac{2s}{t^2+9}$ where $s(0) = e$

101. Find the exact value of $\int_0^\pi \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{3}\right)\ dx$

102. For what values of the parameter $p > 0$ does the following improper integral converge? $\int_0^\infty \frac{1}{t^p}\ dt$ Give a reason for your answer; you must give an explanation to receive credit.

103. The following slope field for the differential equation $\frac{dy}{dx} = f(x,y)$ was drawn on a hand calculator with the window set to $-5 \le x, y \le 5$.



Which of the following functions is most likely to be a solution to the differential equation? Please explain your answer. (If you can rule out some of the answers, include that in your explanation to increase your chances for partial credit.)

a) $y = x^3$ b) $y = e^x$ c) $y = x$ d) $y = x^3 - x$ e) $y = \sin x$

104. Suppose that at 12:00, noon, one summer afternoon, there is a power failure in your home in Tucson, and your cooling does not work. When the power goes out, it is 73° F in your house; the outside temperature is 108°. At 2:00 pm, the temperature in your house has climbed to 85°. Assume that the outside temperature is constant from 12:00 pm until 6:00 pm, and that your house obeys Newton's law of cooling.

a) Write a differential equation that the temperature of your home satisfies.
b) Solve the associated initial value problem.
c) Use your solution to predict the temperature of the house at 6:00 pm.

105. Find a function $y = f(x)$ so that $\frac{dy}{dx} = y\cos^3(x)$ and $f(0) = 1.106$. Give the Taylor series expansion of the following function about $x = 0$. Give as complete an answer as possible including an expression for the n-th term. (The first few terms correctly given may earn partial credit.)

$$f(x) = \int_0^x e^{-t^2}\ dt.$$

107. Find the Taylor expansion of $f(x) = x^2 \sin(x)$ to degree 7 about the point $x = 0$. You must show work to receive credit.

108. Let $f(x)$ be a function so that $f(2) = 6$, $f'(2) = 6$, $f''(2) = 12$, $f'''(2) = 8$ and $f''''(2) = 24$. Find the Taylor approximation of degree 4 of $f(x)$ near $x = 2$.

109. Use the integration formula $\int \frac{dx}{1+\cos x} = \tan(\frac{x}{2}) + C$ to evaluate exactly

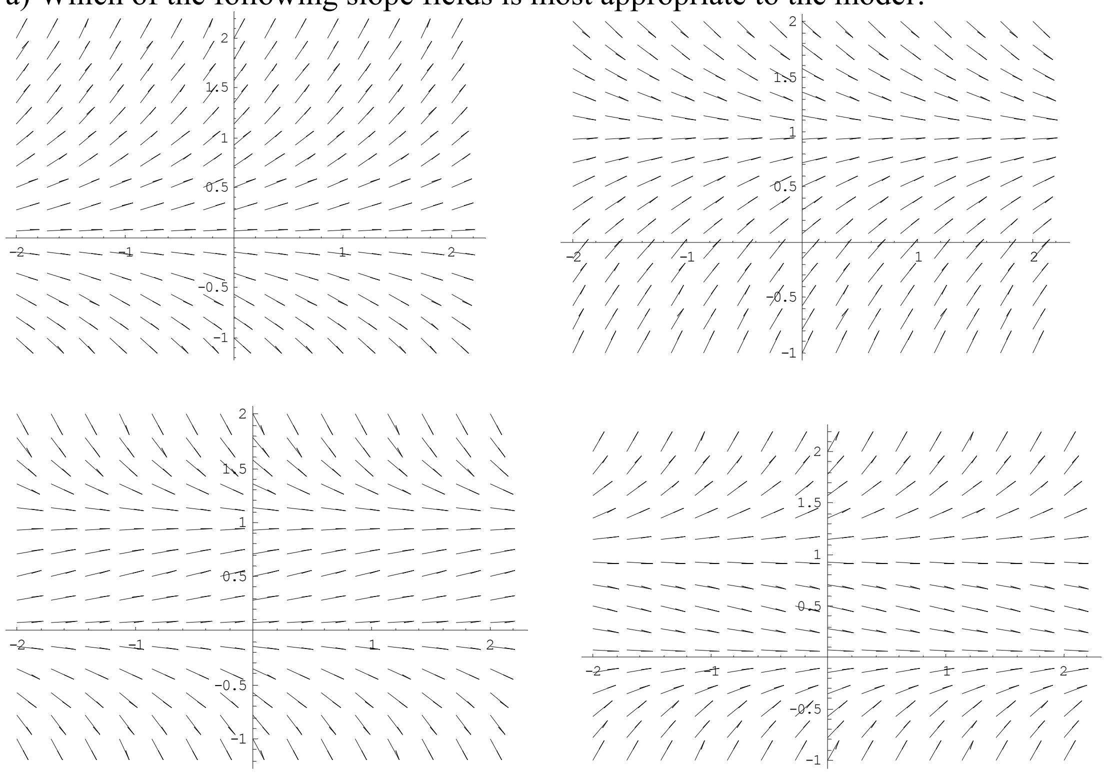
$$\int_0^\pi \frac{1}{4+4\cos(\frac{x}{3})}\ dx$$

110. Find a solution $s(t)$ to the initial value problem:

$$s'(t) = \sin^3(4\,t) \quad \text{where} \quad s(0) = 1$$

111. A social scientist models the spread of a rumor using a differential equation. She will let $P(t)$ stand for the fraction of people who know the rumor at time t. She wants the reasonable properties that if this fraction is ever 0, then it always will be. Also when the fraction is between 0 and 1, then the fraction should grow. Finally if ever it is 1, then the fraction will remain 1. She intends to find a differential equation that models this behavior.

a) Which of the following slope fields is most appropriate to the model?



b) Which of the following differential equations is most appropriate to the model? (Whenever they occur, k and h are positive constants.)

i) $\frac{dP}{dt} = k\,P$ ii) $\frac{dP}{dt} = k\,P\,(1-P)$ iii) $\frac{dP}{dt} = k\,P + h\,(1-P)$

iv) $\frac{dP}{dt} = \frac{k}{P\,(1-P)}$ vi) $\frac{dP}{dt} = k\,P\,(P-1)$ v) $\frac{dP}{dt} = kP(hP-1)$

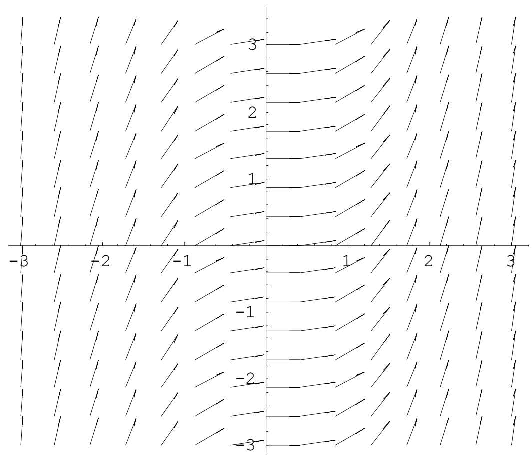
112. Hydrocodone bitartrate is used as a cough suppressant. After the drug is fully absorbed, the quantity of drug in the body decreases at a rate proportional to the amount left in the body. The half life of hydrocodone bitartrate in the body is 3.8 hours. A dose of 10 mg. is administered.

a) Write an initial value problem (a differential equation and a particular value) for the quantity Q of hydrocodone bitartrate in the body at time t measured in hours after the drug is administered.

b) Use the differential equation to find how much of the 10 mg. dose is still in the body after 12 hours.

113. A water tank has the shape of a right circular cone. The top of the tank is a circle with a radius of 8 ft. and the tank has a depth of 15 ft. The tank is filled to a depth of 10 ft. How much work is required to pump all the water out of the tank to a level equal to the top of the tank? (The density of water is 62.4 lbs/ft^3.)
(You must show your work to obtain full credit even for correct answer.)

114 Consider the differential equation $\frac{dy}{dy} = F(x, y)$. Suppose that its slope field is



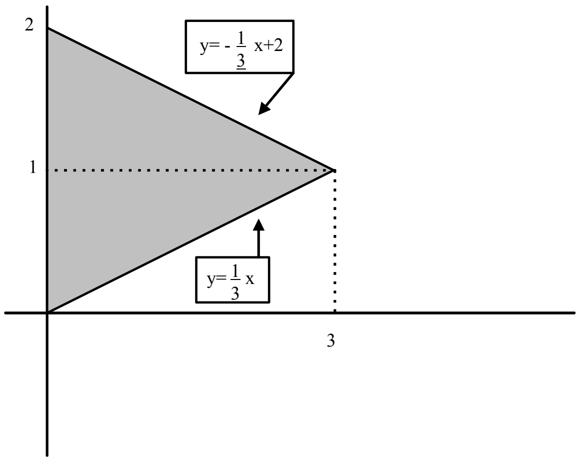
The slope field is representitive of all the features of the differential equation.

Let $f(x)$ be a solution to the differential equation.
Which of the following statements are true.:
a) If $f(-1) = 1$, then $f'(-1) > 0$.
b) No matter what $f(-1)$ is, $f'(-1) > 0$.
c) No matter what x is, $f'(x) \geq 0$.
d) If $f(1) = 1$, then $f''(1) > 0$. (Note the change in x value.)
e) No matter what $f(1)$ is, $f''(1) > 0$.
f) No matter what x is, $f''(x) > 0$.

115. A cylindrical water tank is half filled with water. (The tank is standing on its circular base.) The tank has a radius of 5 ft. and a height of 30 ft. How much work is required to pump the water to a level 6 ft. above the top of the tank? The density of water is
62.4 lbs/ft^3

116. A flag in the shape of an equilateral triangle is hung from the roof of a building. An accident causes the flag to hang from one of its vertices. How much work is necessary to pull the flag back onto the roof? The flag is 16 feet on each side (so its height is $8\sqrt{3}$.) The flag weighs 128 lbs.

117. Find the volume of the solid obtained by revolving the shaded region about the x axis



118. A banner has the shape of a right triangle with sides of 5 ft, 12 ft and 13 ft and it weighs 120 lbs. It is hung from the roof over the side of a building with the shortest side at the top.
a) Set up an integral to compute the work required to lift the banner onto the roof of the building.
b) Evaluate the integral to find the work.

119. A banner in the shape of an isosceles triangle is hung from the roof over the side of a building. As a triangle, the banner has a base of 25 ft. and a height of 20 ft. The banner is made from material with a density of 5 lbs per ft^2. Set up an integral to compute the work required to lift the banner onto the roof of the building. Evaluate the integral to find the work.

120. A banner has the shape of a right triangle with sides of 6 ft, 8 ft and 10 ft and it weighs 12 lbs. (Note: the height of the triangle off the hypotenuse is 4.8 ft.) It is hung from the roof over the side of a building with the hypotenuse at the top. Set up an integral to compute the work required to lift the banner onto the roof of the building. Evaluate the integral to find the work.

121. Consider the initial value problem

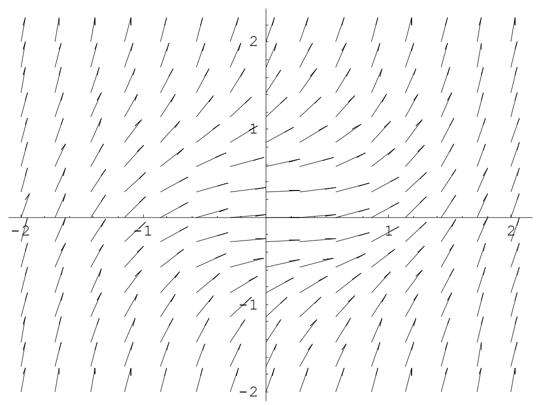
$$\frac{dy}{dx} = 10\,x^2 + y \;; \quad y(0) = 3$$

Use Euler's method to fill in the following table of values for a solution $y = y(x)$. Please give some explanation of your calculations. (There are extra cells in the table for your convenience.)

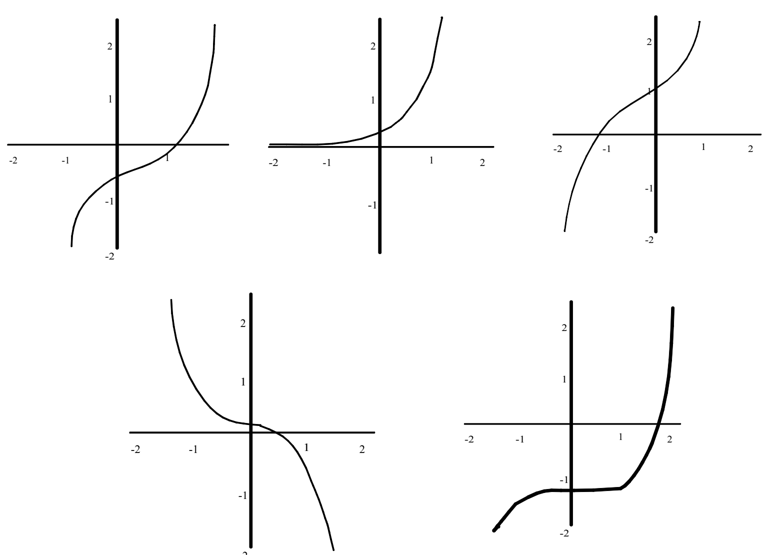
x	0.0	0.1	0.2	0.3
y				

122. Find the volume of the solid whose base is the region in the xy-plane bounded by the curves $y = x^2$ and $y = 8 - x^2$ and whose cross sections perpendicular to the x-axis are squares with one side in the xy-plane.

123. A first order differential equation of the form $\frac{dy}{dx} = F(x, y)$ has the slope field given below.



Which of the following could be the graph of a solution $y = y(x)$ to the differential equation. (There may be more than one.)



124. The rate at which barometric pressure decreases with altitude is proportional to the pressure at that altitude. If the barometric pressure is measured in inches of mercury and the altitude in feet, then the constant of proportionality is 3.7×10^{-5}.

a) Find a differential equation that expresses the relationship described above.

b) Suppose that the barometric pressure at sea level is 29.92 lbs/in^2. What is the pressure outside an airplane flying at 10,000 ft?

125. A particular drug is known to leave a patient's system at a rate directly proportional to the amount in the bloodstream. Previously, a physician administered 9 mg of the drug in a patient, and estimated from a blood test taken 7 hours later that 5 mg remained.
a) Write a differential equation that describes the amount of the drug in the patient's blood stream at time t.
b) Estimate the time when the amount of the drug in the patients blood stream becomes negligible, less than 0.1 mg.

Several weeks later the patient returns, and the physician decides that the drug should be administered a second time. This time the physician administers the drug first with a dose of 5 mg and then continuously using an IV at a rate of 1.25 mg per hour.
c) Write an initial value problem (differential equation and value equality) that describes the amount of the drug in the patient's blood stream at time t under these conditions.(Do not attempt to solve the problem.)

126. Newton's Law of Gravitation says that the force of gravity between two objects of mass M and m a distance x apart is

$$F = \frac{GmM}{x^2}$$

where G is the gravitation constant. Assuming that there are only two objects some distance apart, does it take an infinite amount of work to move one of the objects an infinite distance from the other? Explain your answer completely; a "Yes" or "No" answer will not earn credit unless there is a mathematical explanation.

127. Show your work as you find
a) $\int x^2\, e^x dx$ b) $\int x\, e^{x^2} dx$ c) $\int x\, e^{2x} dx$

128. Match the function with the correct Taylor expansion

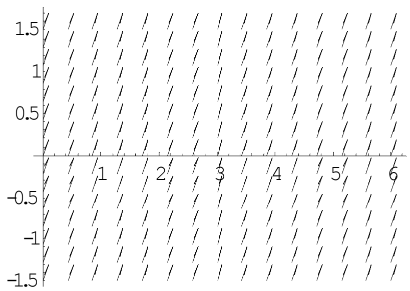
______a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)!}(x-1)^{2k+1}$	i) $x^4 - 9x^3 + 31x^2 - 49x + 31$
______b) $\sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)!}(x-1)^{4k+2}$	ii) sin $(x-1) - (x-1)$
______c) $\sum_{k=0}^{\infty} (x-1)^k$	iii) sin($(x-1)^2$)
______d) $\sum_{k=0}^{4} (-1)^k (x-2)^k$	iv) $\frac{1}{2-x}$

129. Evaluate

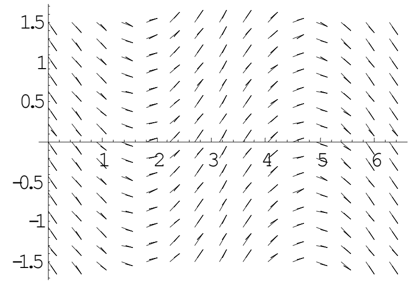
$$\int \frac{e^{2x}}{e^{4x} - e^{2x} - 6}\, dx$$

Hint: Use the substitution $u = e^{2x}$.
You must show each step in the work to receive credit!

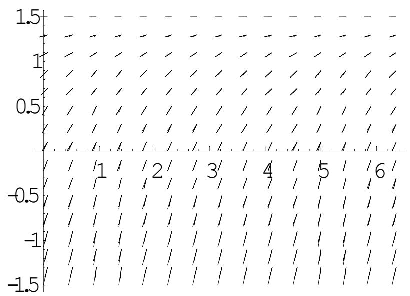
130. A differential equation $\frac{dy}{dx} = f(x, y)$ has $y = \sin(x)$ as a solution. Which of the following slope fields could be the slope field of the differential equation?



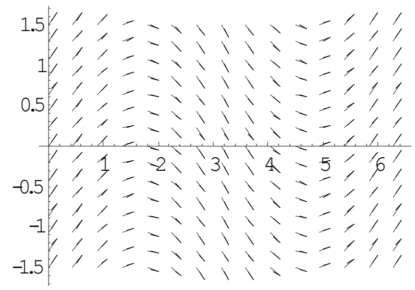
YES NO



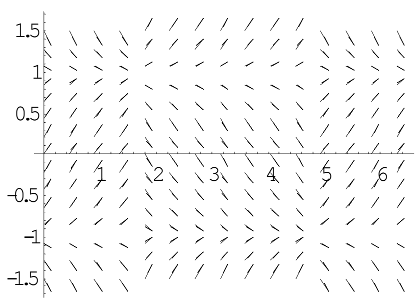
YES NO



YES NO

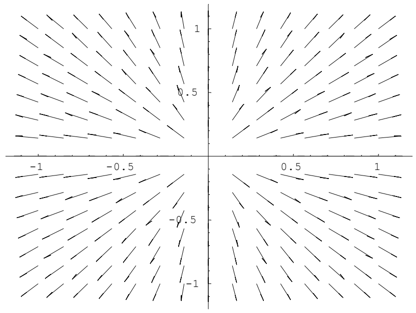


YES NO



YES NO

131. Match the slope field with the differential equation:



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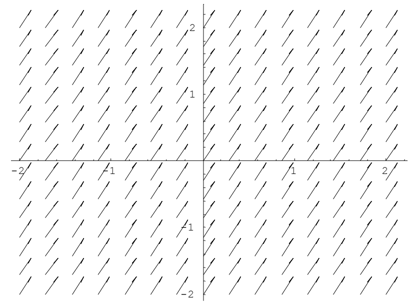

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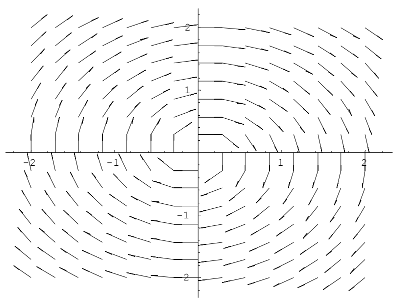

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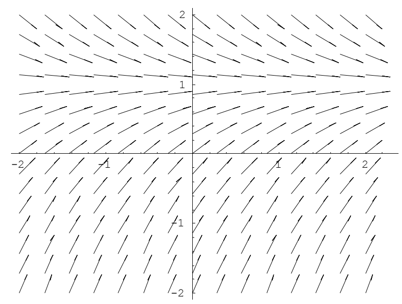

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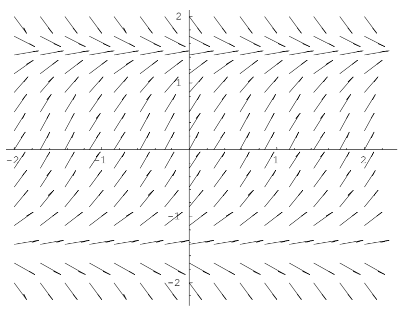


_____

(A) $\frac{dy}{dx} = 2$ (B) $\frac{dy}{dx} = (y-1)$ (C) $\frac{dy}{dx} = (y - \frac{3}{2})(y + \frac{3}{2})$ (D) $\frac{dy}{dx} = -\frac{x}{y}$ (E) $\frac{dy}{dx} = \frac{y}{x}$

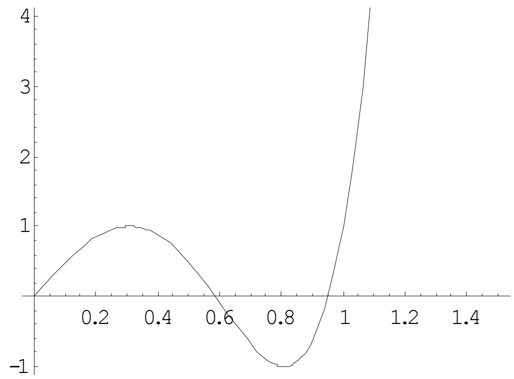
132 An engineer estimates that the amount of work necessary for a certain task is given by

$$\int_3^\infty \frac{1}{x \ln x}\, dx$$

Does this require a finite or an infinite amount of work?

YOU MUST JUSTIFY YOUR ANSWER TO RECEIVE ANY CREDIT.

133. There are a number of functions in Mathematics named after the Russian Mathematician Chebyshev. One is usually written as $T_5(x)$. Its domain is all real numbers. A part of its graph, for x from 0 to 1.5, looks like



True or False: Circle the correct answer

a) $\int_0^{0.1} T_5(x)\,dx \le \int_0^{0.2} T_5(x)\,dx$ $\quad TRUE \quad FALSE$

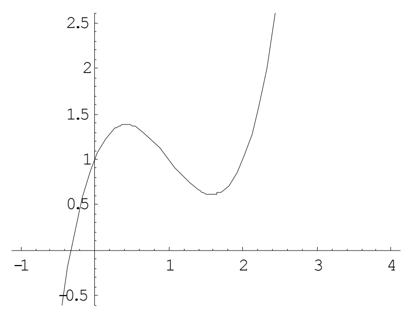
b) $\int_0^{0.4} T_5(x)\,dx \le \int_0^{0.5} T_5(x)\,dx$ $\quad TRUE \quad FALSE$

c) $\int_0^{0.1} T_5(x)\,dx \le \int_0^{0.1} (T_5(x))^2\,dx$ $\quad TRUE \quad FALSE$

d) $\int_0^1 T_5(x)\,dx \ge 0$ $\quad TRUE \quad FALSE$

e) $\int_0^1 |T_5(x)|\,dx \ge 1$ $\quad TRUE \quad FALSE$

134. The graph of the function $f(x) = x^3 - 3x^2 + 2x + 1$ is



If the region bounded by this curve, the x-axis, $x = 0$ and $x = 2$ is revolved about the x-axis, what is the volume of the resulting solid? Give exact volume for full credit.

135. A cylindrical barrel, standing upright on its circular end, contains muddy water. The top of the barrel, which is open, has a diameter of 1 meter. The height of the barrel is 1.8 meters, and the depth of the water in the barrel is 1.5 meter. The density of the muddy water varies with the depth of the water, and is given by

$$\rho(h) = (1 + k\,h)\,\frac{\text{kg}}{\text{m}^3}$$

where h is the depth measured as the distance to the surface (from the top to the bottom), and k is a positive constant. Find the work necessary to pump the muddy water to the top rim of the barrel. (You may leave constants like k, π and g (the acceleration due to gravity) in your answer unevaluated.)

136. Solve $\frac{dy}{dx} = -\frac{x}{y}$ where $y(1) = 2$ and describe the graph of the solution.

137. An object moves along the real line. Let $s(t)$ be the position of the object at time t seconds. Match different assumptions about the motion of this object with the Differential Equation that correctly reflects the assumptions. Throughout, k, l and n are taken as positive constants.

___	The velocity of the object is directly proportional to the time it has been in motion.	a) $s''(t) = k$
___	The acceleration of the object is directly proportional to the time it has been in motion.	b) $s'(t) = k\,t$
___	The velocity of the object is directly proportional to its position	c) $s'(t) = k\,s(t)$
___	The acceleration of the object is directly proportional to its position	d) $s''(t) = k\,s(t)$
___	The acceleration of the object is a linear function of its velocity.	e) $s''(t) = k\,t$
___	The acceleration of the object is a linear function of its velocity and its position.	f) $s''(t) = k\,s'(t) + l$
___	The acceleration of the object is constant.	g) $s''(t) = k\,s'(t) + l\;s(t) + n$

138. Do these improper integrals converge or diverge:

a) $\int_0^1 \frac{1}{x^2}\,dx$ Converges Diverges

b) $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$ Converges Diverges

c) $\int_1^\infty \frac{x^2+2x}{\sqrt{x^3+3x^2+x}}\,dx$ Converges Diverges

d) $\int_0^1 \frac{x^2+2x}{\sqrt{x^3+3x^2+x}}\,dx$ Converges Diverges

139. Let

$$f(x) = \sum_{k=0}^{\infty} \frac{k+1}{7} x^k = \frac{1}{7} + \frac{2}{7}x + \frac{3}{7}x^2 + \frac{4}{7}x^3 + \frac{5}{7}x^4 + ...$$

Find an exact value for $\int_0^{\frac{1}{13}} f(x)\,dx$. (An approximation accurate to 5 decimal places will earn partial credit.)

140. Give the series expansions of the following functions **include the radius of convergence with your answer:**

a) $f(x) = \sin(x)$
b) $f(x) = \cos(x)$
c) $f(x) = \frac{1}{1-x}$
d) $f(x) = e^x$

141. Find $\int \frac{\sin(3x)}{\cos^2(3x)-1}\,dx$ using the substitution $w = \cos(3x)$.

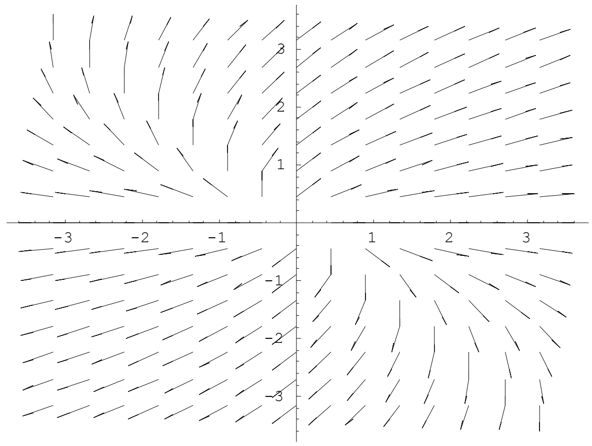
142. Let $f(x) = \sum_{k=0}^{\infty} \frac{k(k-1)(k-2)}{k!}\, x^k$ and $g(x) = \sum_{k=0}^{\infty} (2k+5)\, x^{k+3}$.

a) Write out the Taylor series of $f(x)$ out to x^7.
b) Write out the Taylor series of $g(x)$ out to x^7
c) Write out the Taylor series of $f(x)g(x)$ out to x^7
d) Find $\lim_{x\to 0} \frac{g(x)}{f(x)}$

143. Solve the following initial value problem.

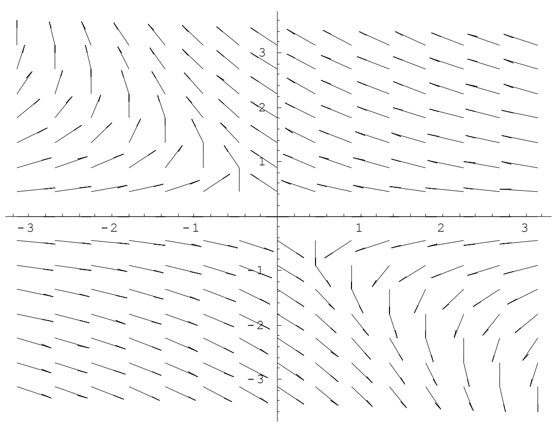
$$\frac{dy}{dx} = \frac{x(y-1)}{x^2+9} \text{ where } y(4) = 7$$

144. The differential equation $\frac{dy}{dy} = f(x, y)$ has slope field

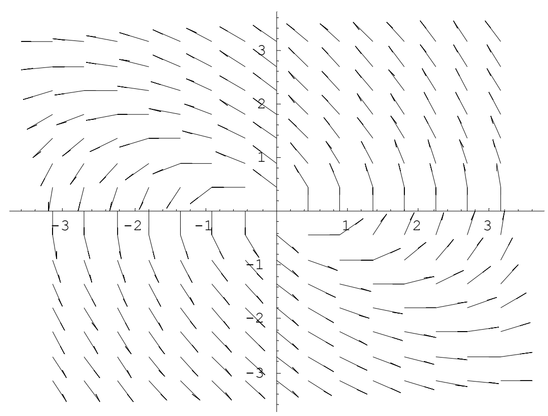


Match the related differential equation with its slope field

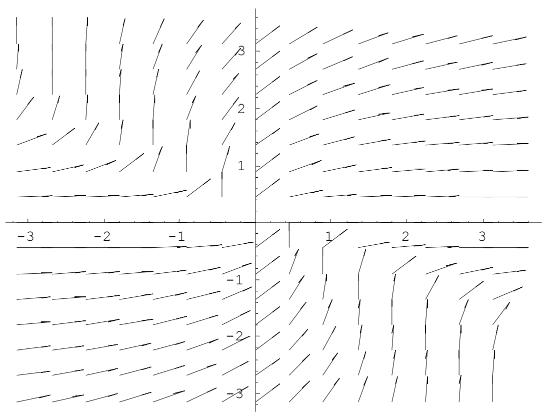
A: $\frac{dy}{dx} = -f(x, y)$ **B:** $\frac{dy}{dx} = -\frac{1}{f(x,y)}$ **C:** $\frac{dy}{dx} = (f(x, y))^2$ **D:** $\frac{dy}{dx} = f(x, -y)$



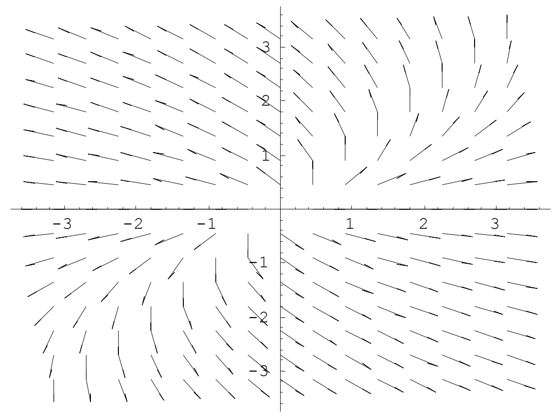
Equation ________



Equation ________



Equation ________

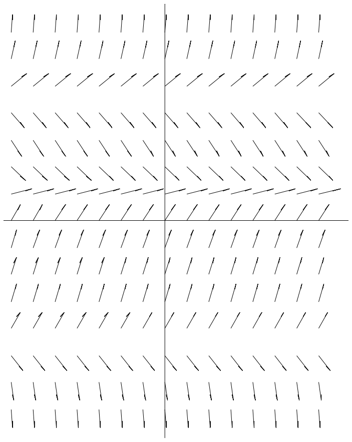


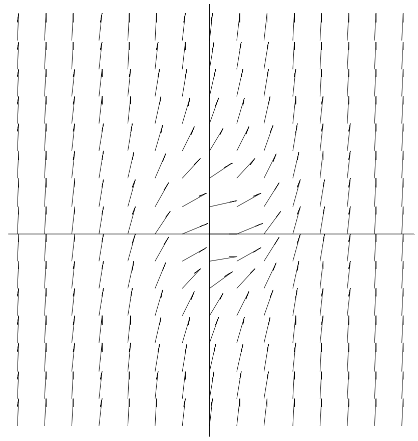
Equation ________

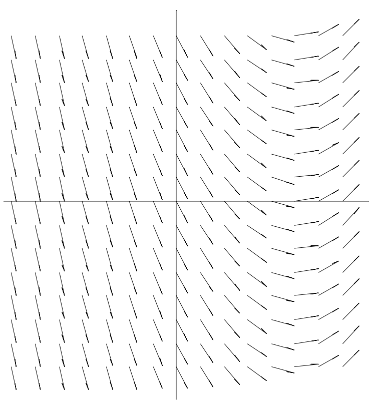
145. Match the differential equation with the proper slope field. Throughout a is a positive constant.

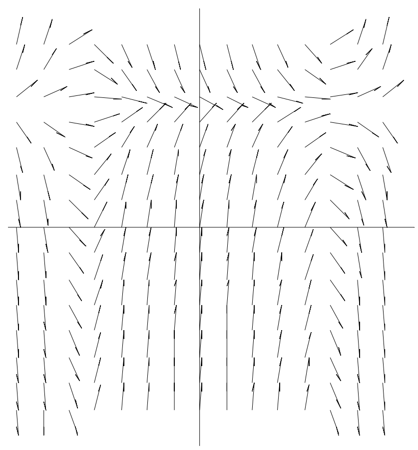
a) $\frac{dy}{dx} = ax^2 + y^2$ b) $\frac{dy}{dx} = (y^2 - 4)(y - a)$ c) $\frac{dy}{dx} = (x^2 - 4)(y - a)$

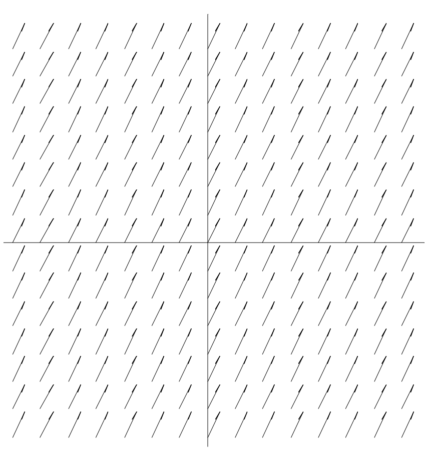
d) $\frac{dy}{dx} = a$ e) $\frac{dy}{dx} = x - a$ f) $\frac{dy}{dx} = y + a$

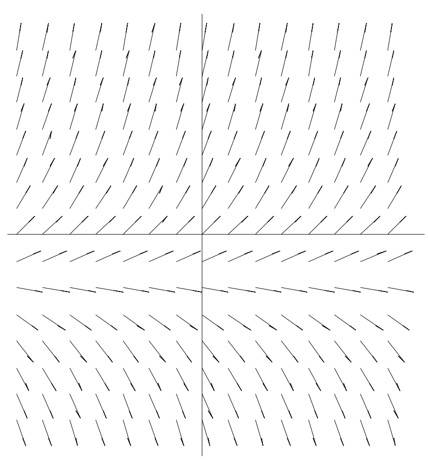








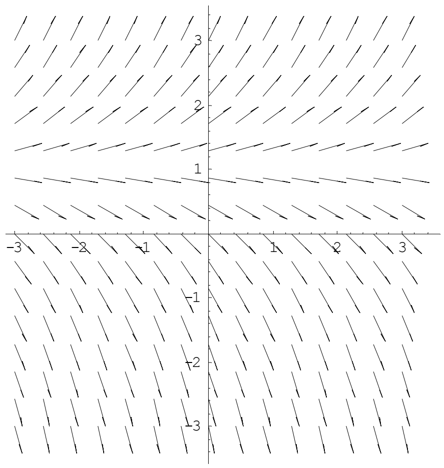


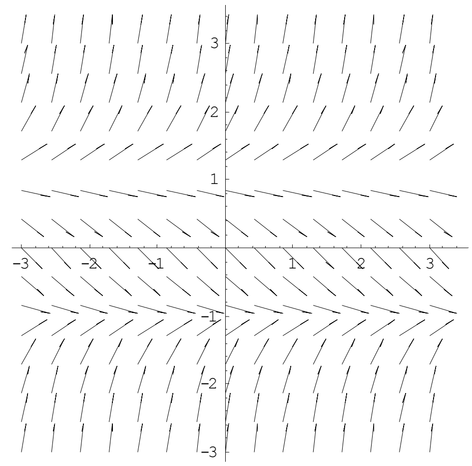


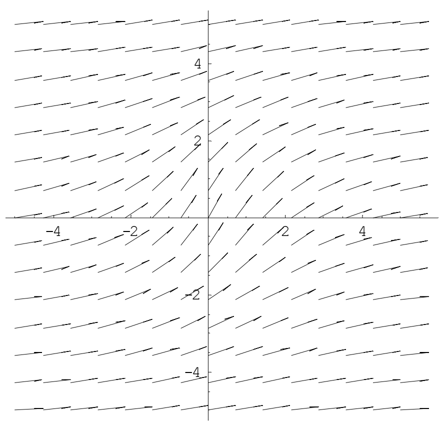
146. Match the differential equation to the slope field.

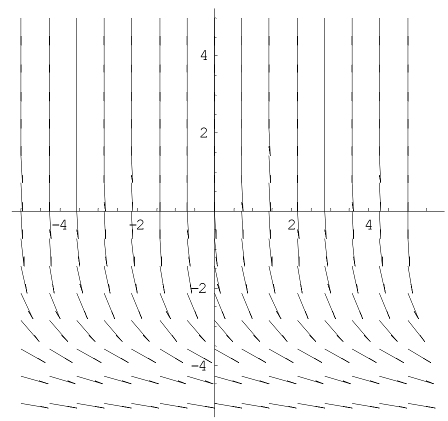
I. $\frac{dy}{dx} = y - 1$ II. $\frac{dy}{dx} = (y^2 - 1)$ III. $\frac{dy}{dx} = \frac{1}{x^2+y^2+3}$

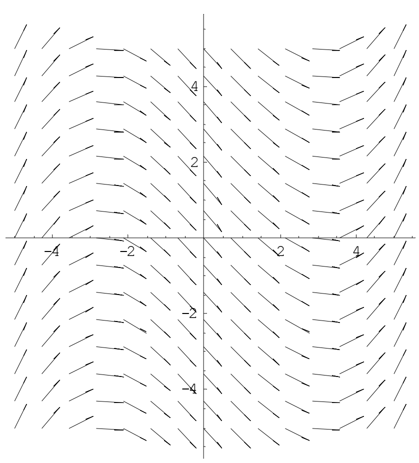
IV. $\frac{dy}{dx} = -e^{y+3}$ V. $\frac{dy}{dx} = \frac{1}{8}(x^2 - 9)$ VI. $\frac{dy}{dx} = |y|$

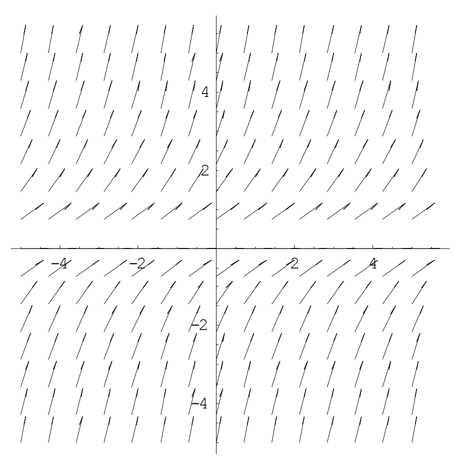












147. Find: a. $\int \sin\left(\frac{x}{2}\right)\ dx$ b. $\int e^{-3t}\ dt$ c. $\int \frac{2}{3y+1}\ dy$

d. $\int \frac{3}{\theta^2+81}\ d\theta$ e. $\int \frac{x^2+1}{x}\ dx$

148. A banner in the shape of an isosceles triangle is hung from the roof over the side of a building. As a triangle, the banner has a base of 35 ft. and a height of 30 ft. The banner is made from material with a density of 5 lbs per ft^2. Set up an integral to compute the work required to lift the banner onto the roof of the building. Evaluate the integral to find the work.

149. A tank is filled to capacity with 100 gal. of pure water. A salt solution of 2 lbs./gal. is pumped into the tank at a rate of 5 gal./min. Assume that the water in the tank mixes immediately and that excess water is drained off at the same rate at which *salt* water is added.

Set up an initial value problem for $A(t)$, the amount of salt in pounds, in the tank at time t. Do not solve the problem.

150. Find the volume of the solid obtained by revolving the region above the graph of $y = x^2$ and below the line $y = 9$ about the line $y = 9$. Give an exact answer and show your work.

151. Match the function with its Taylor expansion:

______. $f(x) = \sin(x) + \cos(x)$	a) $2 + x^2 + \frac{1}{12}x^4 + \frac{1}{360}x^6 + \ldots\ldots\ldots$
______. $f(x) = \cos(x) - \sin(x)$	b) $2 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots\ldots$
______. $f(x) = e^x + e^{-x}$	c) $1 + x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4 + \frac{1}{24}x^5 + \ldots\ldots$
______. $f(x) = 2 + \ln(1+x)$	d) $1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \ldots$
______. $f(x) = 1 + x\,e^{-x}$	e) $1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \ldots$