Math 124 and 125
Past Exam Questions

Here are some questions that appeared on common exams in past semesters. This is not a sample exam, but it is a reasonable guide to the style and level of common exam given by the U of A Mathematics Department.

1. Find the derivatives of the following functions:
a) $f(x) = \sin(2x+3)$ b) $G(t) = \cosh(t^2)$ c) $h(t) = \frac{\sin(t)}{t}$ d) $g(t) = t \ln(t)$

2. Evaluate
a) $\int \sin(2x)\ dx$ b) $\int \cos(\frac{x}{2})\ dx$ c) $\int 2e^{2x}\ dx$ d) $\int \frac{x+2}{x}\ dx$

3. Find $\frac{dy}{dx}$ where $y = e^{2x} + e^{x^2} + e\,x$.

4. Find $\frac{dy}{dx}$ where $y = (\frac{\pi}{2}x + \frac{\pi}{2})^5 + \sin(x + \frac{\pi}{2}) + \cos(\frac{\pi}{2}x)$.

5. Find $\frac{dy}{dx}$ where $y = \sin^2 x + \sin 3x + \sin(x^4)$

6. Find $\frac{dy}{dx}$ where $y = \frac{1}{\sin(2x) + 2\cos x}$

7. For each of the following find $\frac{dy}{dx}$. (No partial credit; so be careful.)
a) $y = \sqrt{4x+7}$ b) $y = \arctan(10\,x)$ c) $y = \cos(\frac{x}{2})$
d) $y = e^x + e^{-x}$ e) $y = \frac{x}{x+1}$

8. Find the second derivative of $f(x) = \sqrt{x^2 + a^2}$. Simplify your answer. Here a is a nonzero constant.

9. Match the function with its derivative:

_____ $f(x) = \arctan(x^2)$	A. $f'(x) = \frac{-2x}{\sqrt{1+x^2}+x^2\sqrt{1+x^2}}$
_____ $f(x) = 4\arctan(x^{\frac{1}{2}})$	B. $f'(x) = \frac{2x}{1+x^2}$
_____ $f(x) = \frac{2}{\sqrt{1+x^2}}$	C. $f'(x) = \frac{2x}{1+x^4}$
_____ $f(x) = \ln(1+x^2)$	D. $f'(x) = \frac{2x^3}{1+x^4}$
_____ $f(x) = \ln(\sqrt{1+x^4})$	E. $f'(x) = \frac{2}{(x\sqrt{x}+\sqrt{x})}$

10. Find the exact value of the following, you must show your work for credit.
a) $\int_0^1 \sin(\pi x)\,dx$
b) $\int_{-1}^1 (3 + x + 2x^3)\,dx$

11. Find an equation of a line tangent to the curve $y = x^2 - 4x + 3$ that has a slope of 6.

12. Find the exact value of

a) $\lim\limits_{t \to 0} \frac{\sin(3t)}{t}$ b) $\lim\limits_{x \to \infty} \frac{3x^2 - 2x + 4}{7x^2 - 1}$

c) $\lim\limits_{x \to \pi} \frac{\sin(x)}{x}$ d) $\lim\limits_{\theta \to \infty} \arctan(\theta)$

13. Find the exact value of $f''(2)$ where $f(x) = \arctan(x)$

14. a) Find the equation of the line tangent to the curve $y = x^4 - x^3 + x^2 - x$ at $x = 55$.
b) Is this line above or below the curve near the point?

15. Consider the function $y = f(x)$ near the point $(-2, 3)$ given implicitly by

$$y^3 + 2x^2y + x^2 - 7x = 69$$

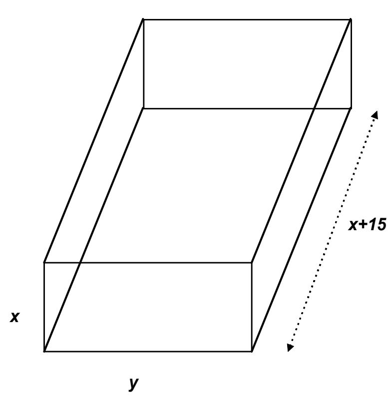
a) Find $\frac{dy}{dx} = f'(-2)$ b) Find the equation of the tangent at $(-2, 3)$.
c) Does this tangent line hit the curve $y^3 + 2x^2y + x^2 - 7x = 69$ in a point other than the point of tangency?

16. Let $f(x) = \frac{1}{1 + e^{\frac{1}{x}}}$. Do the following limits exist?

a) $\lim\limits_{x \to \infty} f(x)$ b) $\lim\limits_{x \to 1} f(x)$ c) $\lim\limits_{x \to 0^+} f(x)$

d) $\lim\limits_{x \to 0^-} f(x)$ e) $\lim\limits_{x \to 0} f(x)$

17. Wire with a total length 420 inches will be used to construct the edges of a rectangular box and thus provide a framework for the box. One of the edges must be exactly 15 inches longer than one other edge. What is the maximum possible volume that such a box can have? The exact answer will earn full credit, and you must show your work to receive credit for an approximation.



18. Find the equation of the line tangent to the curve given by the equation

$$y^2 + \cos^2(2x) = 1$$

at the point $(\frac{5\pi}{12}, \frac{1}{2})$. Give an exact answer.

19. Match the curve with the correct parametrization. Note the differences in the scales used on the x and y axes.

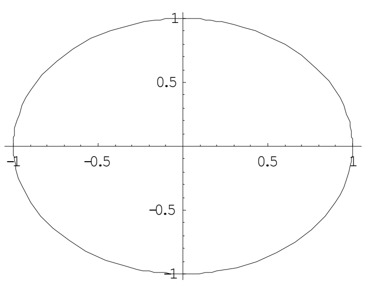
A. $x = t$
$y = t^3 - 12t + 1$
$-5 \leq t \leq 5$

B. $x = t^3 - 12t + 1$
$y = t^3 - 12t + 1$
$-4 \leq t \leq 4$

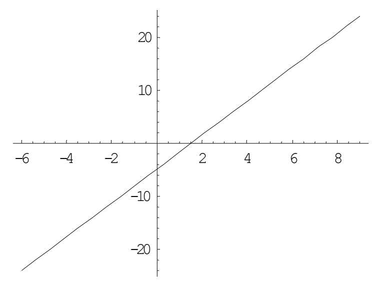
C. $x = \cos(t)$
$y = \sin(t)$
$0 \leq t \leq 2\pi$

D. $x = 5t - 1$
$y = 16t - 8$
$-1 \leq t \leq 2$

E $x = 5t - 1$
$y = -6t + 8$
$-2 \leq t \leq 2$



Answer ______



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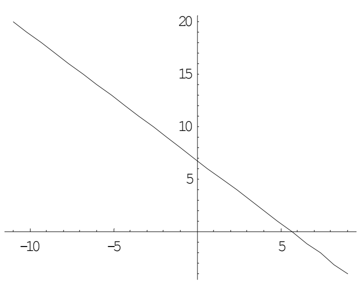

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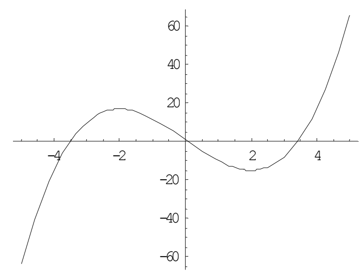
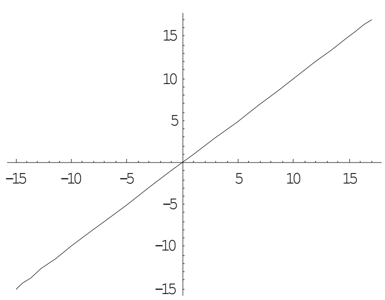


Answer ________



Answer_____

20. a) Find $\frac{dy}{dx}$ where $y^3 + xy = 20$.
b) Find the derivative of $g(u) = \sin(\ln u)$.

21. Let a be a constant. For each of the following, find $\frac{dy}{dx}$:
a) $y = \sin(ax)$ b) $y = a\sin(x)$
c) $y = (\sin(x))^a$ d) $y = (\sin(x^a))$

22. What is the exact point on the curve parametrized by

$$x(t) = t^4 - 20\,t - 3$$
$$y(t) = t^3 + 3$$

that is furthest to the left on the plane?

23. Evaluate
a) $\int \frac{x+b}{x}\, dx$ b) $\int \frac{x}{x+b}\, dx$

c) $\int \frac{b}{x^2+1}\, dx$ d) $\int \frac{x}{bx^2+b}\, dx$

24. Find:
a) $\int \frac{2x+1}{2x}\, dx$ b) $\int \sin(x)\, dx$ c) $\int_{-1}^{2} (t^2 - 2t)\; dt$ d) $\int_0^1 \frac{dr}{r^2+1}$

25. Find the equation of the line tangent to the curve $y = (\sin x)(\cos x)$ at the point $(\frac{\pi}{4}, \frac{1}{2})$.

26. Find the equation of the line tangent to the graph of $y = 3\,e^{2x} - e^{-3x}$ at the point $(0, 2)$.

27. Find the equation of the line tangent to $y = \cos(\pi x)$ at $x = \frac{1}{2}$. Give exact values in your answer.

28. Consider the graphs of the following functions. Which have at least one tangent line of every possible **positive** slope? Explain your answer. (To have at least one tangent line of every possible **positive** slope, there does not need to be a point where the slope of the tangent is 0, but every positive number should appear as the slope of a tangent somewhere.)
a) $f(x) = \sin(x)$ b) $f(x) = x^{31} + x + 9$
c) $f(x) = e^x$ d) $f(x) = \arctan(x)$ e) $f(x) = \ln(x)$

29. Find the equation of the line tangent to the curve $y^3 + y^2 + xy + x^2 = 15$ at the point (1,2).

30. On close inspection, you will see that the graph of the function $f(x) = \frac{(3x-5)(x-3)}{x^4}$ is below the x-axis for certain values of x. Find the exact area of the finite region in the plane that is between the graph of $f(x)$ and the x-axis. You must show your work for full credit.

31. Find the derivative of

$$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 1 & \text{if } x = -1 \\ x^2 & \text{if } -1 < x < 2 \\ 4 & \text{if } x = 2 \\ 4x-4 & \text{if } 2 < x \end{cases}$$

(If the derivative does not exist, you must indicate this.)
Answer:

$$f'(x) = \begin{cases} \qquad\qquad & \text{if } x < -1 \\ & \text{if } x = -1 \\ & \text{if } -1 < x < 2 \\ & \text{if } x = 2 \\ & \text{if } 2 < x \end{cases}$$

32. Let $y = x^3 - 2x + 1$.
a) Find the equation of the line tangent to the curve at $(a, a^3 - 2a + 1)$.
b) For what values of a is the tangent line above the curve near the point of tangency and at what values of a is the tangent line below the curve near the point of tangency?

33. A function $y = f(x)$ satisfies the equation $y^2 = \frac{x^2}{xy-4}$. Find the slope of the tangent line to $y = f(x)$ at $(4, 2)$

34. Find the equation of the line tangent to the curve $y = \frac{x}{x^2-3}$ at the point $(2,\ 2)$.

35. Evaluate $\int_0^4 x\sqrt{x^2+9}\ dx$ exactly.

36. Find the following indefinite integrals

a) $\int \frac{\sin x}{\cos^3 x}\ dx$ b) $\int \frac{e^{2x}}{e^{2x}+1}\ dx$

37. Find the exact area enclosed by the graph of $y = 2 - e^x$, the x-axis and the y-axis.

38. Evaluate the exact value of following integrals:

a) $\int_{-1}^{1} (6 - x - x^6)\ dx$ b) $\int_0^2 x^2\sqrt{x^3+1}\ dx$

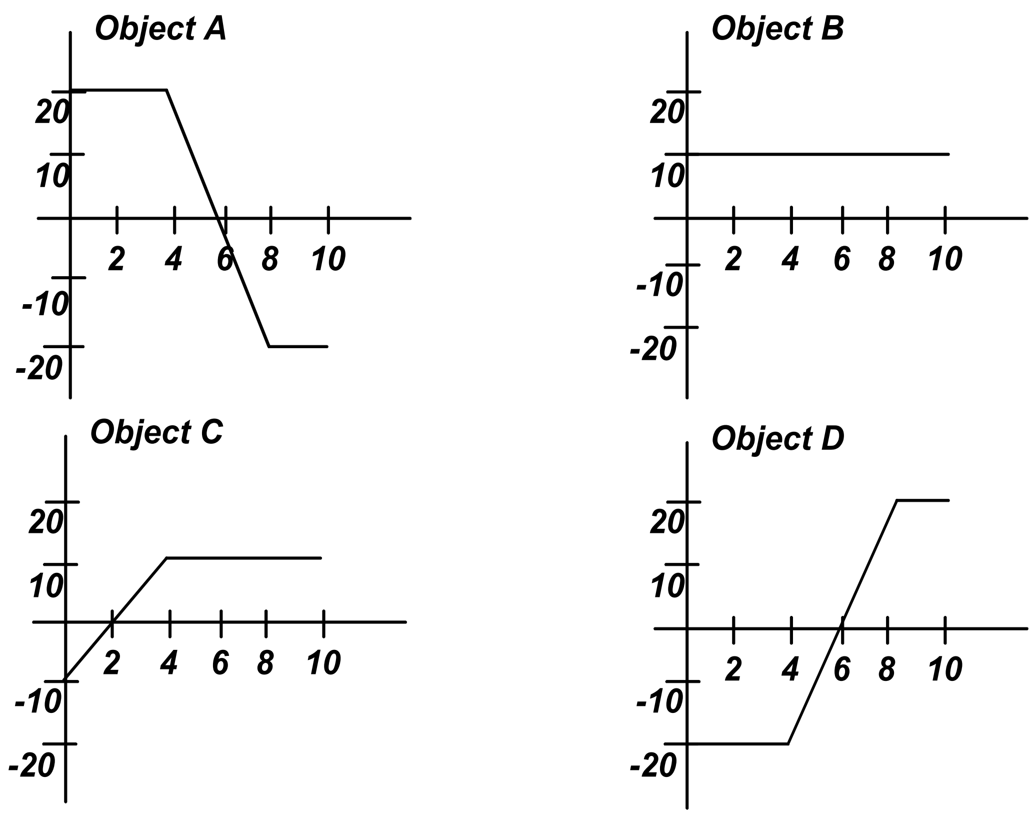
39. Find the area enclosed by the curve $y = 16 - x^2$ and the x-axis.

40. a) Find $\int e^{-2x}\ dx$

b) Evaluate the following definite integral; give an exact answer $\int_{-2}^{4} (3r^2 - 2r)\ dr$

41. Several objects are moving along a straight line from time $t = 0$ to time $t = 10$. The following are graphs of the **velocities** of these objects.
a) Which object is farthest from its original position at the end of the 10 seconds?
b) Which object is closest to its original position at the end of the 10 seconds?
c) Which object has traveled the greatest total distance during these 10 seconds?
d) Which object has traveled the least total distance during these 10 seconds?

VELOCITY AT TIME t



42. A function $f(x)$ is approximated in the table below.

x	0.0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.1	3.2	3.4	3.6	4.0	4.4

(The values in the table are representative of the properties of the function.)

Use the table to approximate $\int_0^1 f(x)\, dx$

43. Find the exact area enclosed by the curve $y = e^x$ and the x-axis and the lines $x = -1$, and $x = 1$. (Approximations will **not** earn partial credit.)

44. Find the exact area enclosed by the curve $y = 6 + x - x^2$ and the x-axis.

45. Find the exact equation of one tangent line to the curve $y = \sin x$ with slope $\frac{1}{2}$.

46. A function $f(x)$ is approximated in the table below .

x	1.00	1.50	2.00	2.50	3.00	3.50	4.00
$f(x)$	1.12	1.06	1.02	1.0	1.03	1.15	1.65

(The values in the table are representative of the properties of the function.)

Use this table to approximate the values in the following table of the derviative:

x	1.25	1.75	2.25	2.75	3.25	3.75
$f'(x)$						

47. Order the following numbers from **smallest to largest**:
$A = \int_0^1 e^{x^2}\,dx$; $B = \int_1^2 e^{x^2}\,dx$; $C = \int_2^1 e^{x^2}\,dx$; $D = \int_{-3}^{-2} e^{x^2}\,dx$;
$E = \int_{-1}^{-1} e^{x^2}\,dx$; $F = 1$

48. Sketch a smooth curve $y = f(x)$ on $0 \le x \le 10$ satisfying the following:

- $f(x)$ takes on the values -2 and 2 somewhere on $0 \le x \le 10$
- $f'(x) < 0$, for all x
- $f''(x) > 0$, for all x.

Point out reasons that your curve meets the conditions.

49. A function $f(x)$ is continuous and differentiable, and has values given in the table below.

$\boldsymbol{x}$	**3.0**	**3.25**	**3.5**	**3, 75**	**4.0**
$\boldsymbol{f(x)}$	**1.34**	**1.5**	**2.0**	**2.5**	**2.85**

(The values in the table are representative of the properties of the function.)

Suppose $l(x)$ is the linear function with graph tangent to the graph of $f(x)$ at $z = 3.5$. Fill in the values for the tangent $l(x)$.

$\boldsymbol{x}$	**3.0**	**3.25**	**3.5**	**3, 75**	**4.0**
$\boldsymbol{l(x)}$					

50. Consider a function $y = f(x)$ and its derivative $f'(x)$. Suppose that values of the derivative function are given in the following table :

x	10	10.5	11	11.5	12	12.5	13
$f'(x)$	1	2	1	0.5	0	-1	-2

(The values in the table are representative of the properties of the function.)

Use it to estimate the values of the function missing in the following table

x	10	10.5	11	11.5	12	12.5	13
$f(x)$	3						

51. A function $f(x)$ is continuous and differentiable, and has values given in the table below.

x	2.0	2.2	2.4	2.6	2.8
$f(x)$	5.0	5.5	5.9	6.3	6.8

(The values in the table are representative of the properties of the function.)

Suppose $l(x)$ is the linear function with graph tangent to the graph of $f(x)$ at $z = 2.4$. Fill in the values for the tangent $l(x)$.

x	2.0	2.2	2.4	2.6	2.8
$l(x)$					

52. A heavy object is dropped off the side of a large cliff. It takes 4 seconds to hit the ground. Let a be the acceleration on the object, v its velocity, s the distance fallen, and t the time since it was dropped.
a) Sketch a graph of the acceleration, a, against time t.
b) Sketch a graph of the velocity, v, against time t
c) Sketch a graph of the distance, s, against time t
d) Sketch a graph of the distance, s, against velocity v

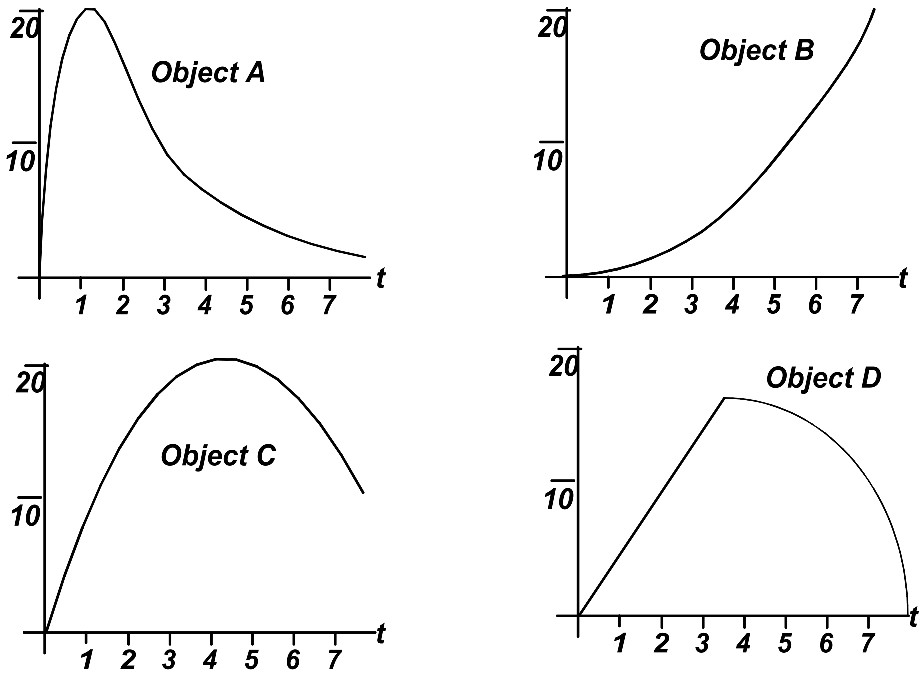
53. Find the equations to all the lines with a slope of 12 that are tangent to the graph of

$$y = x^3.$$

Give exact answers, and draw a graph that illustrates your answer.

54. Let $y = x^3 - 2x + 1$.
a) Find the equation of the line tangent to the curve at $(a, a^3 - 2a + 1)$.
b) For what values of a is the tangent line above the curve near the point of tangency and at what values of a is the tangent line below the curve near the point of tangency?

55. The following graphs show the distance of four objects from their starting point at times between 0 and 7 with time is measured in seconds.



a.) Which object ends up furthest from its starting point?
b) Which object starts off at the highest speed?
c) Which objects accelerate at some point in the first 7 seconds?
d) Which objects decelerate at some point in the first 7 seconds?
e) Which objects, if any, hold their position fixed for some period during the first 7 seconds?
f) Which objects, if any, hold their velocity fixed for some period during the first 7 seconds?
i) Which objects, if any, have a zero velocity at least instantaneously during the first 7 seconds?
j) Which objects, if any, have a zero acceleration at least instantaneously during the first 7 seconds?

56. Let $f(x)$ and $g(x)$ be functions. Suppose both functions are differentiable for $-10 \le x \le 10$. Let

$$f(-1) = 3;\quad f'(-1) = 6;\qquad g(-1) = 3;\quad g'(-1) = -11$$
$$f(3) = 5;\quad f'(3) = -3;\qquad g(3) = 2;\quad g'(3) = -4$$

Consider the functions:

$$p(x) = f(x)\cdot g(x);\qquad q(x) = \frac{f(x)}{g(x)}\qquad \text{and}\qquad h(x) = g(f(x))$$

Compute:

a) $h(-1) =$ b) $p(-1) =$ c) $q(-1) =$
d) $h'(-1) =$ e) $p'(-1) =$ f) $q'(-1) =$

57. Find: (You must show work or give an explanation for credit.)

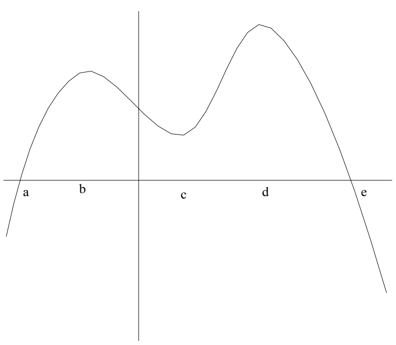
a) $\lim_{t\to 0}\frac{\sin(t)}{t}$ b) $\lim_{t\to 0}\frac{\sin(3t)}{t}$ c) $\lim_{t\to 0}\frac{\sin^2(t)}{t}$ d) $\lim_{t\to 0}\frac{\sin(2t)}{\sin(7t)}$

58. Several objects start the same location and move along a straight line from time $t = 0$ to time $t = 20$ seconds. The following formula give the **velocities** of these objects.

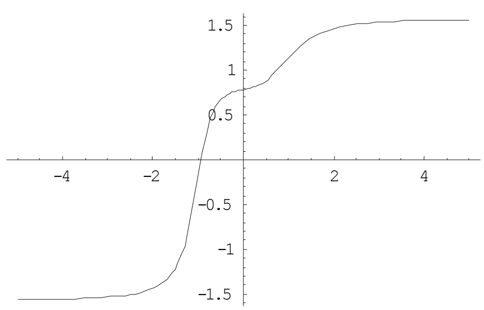
Object 1: $v_1(t) = 10 - t$
Object 2. $v_2(t) = 10 - 2t$
Object 3: $v_1(t) = 10 - \frac{1}{2}t$
Object 4. $v_2(t) = |\,10 - t\,|$

a) Which object is farthest from its original position at the end of the 20 seconds?
b) Which object is closest to its original position at the end of the 20 seconds?
c) Which object has traveled the greatest total distance during these 20 seconds?
d) Which object has traveled the least total distance during these 20 seconds?

59. Sketch a graph of $y = f'(x)$ where $y = f(x)$ has the graph



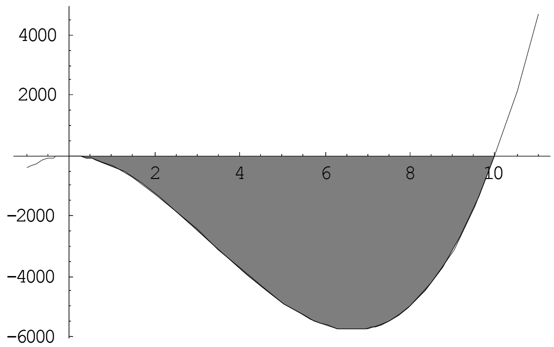
60. Consider the function $y = f(x)$ given by the graph



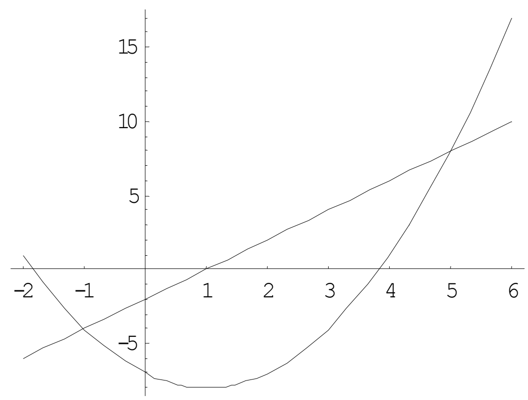
Answer the following question giving an explanation of your answer:

a) Is $g(x) = (f(x))^2$ an increasing function?
b) Is $h(x) = (f(x))^3$ an increasing function?
c) Is $e(x) = \frac{1}{f(x)}$ a decreasing function?
d) Is $d(x) = f'(x)$ an increasing function?

61. Find the exact area bounded by the x-axis and the graph of $y = 39\,x^3 - 390\,x^2$. You must show your work!



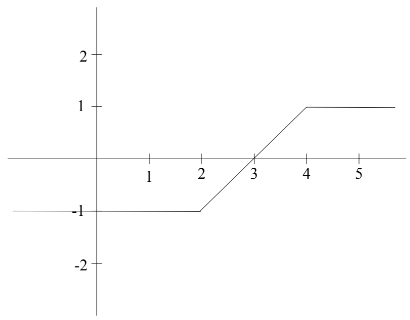
62. The graph of the function $f(x)$ and its derivative $f'(x)$ are given in the figure. Use this information to find **the values of $\boldsymbol{x}$** that maximize and minimize the function $g(x) = f(x)\,e^{-x}$



63. An object is brought from rest to a speed of 6 meters per second in exactly 5 seconds under constant acceleration. Let a be the acceleration on the object, v be the velocity of the object, s be the distance traveled, and t be the time since the acceleration began.
All graphs should be fully labeled and accurate.
a) Sketch a graph of the acceleration, a, against time t with $0 \leq t \leq 5$.
b) Sketch a graph of the velocity, v, against time t with $0 \leq t \leq 5$.
c) Sketch a graph of the distance s against time t with $0 \leq t \leq 5$.

64. A farmer wishes to build a fenced coral with a feeding trough through the center. Fencing material costs \$45 per foot while it costs \$153 per foot of length to build the feeding trough. If the coral must have an area of 8100 ft.^{2} on each side of the trough, what is the lowest possible total cost? (The width of the feeding does not matter to the cost.)

65 Consider the function $f(x)$ given by the graph below:

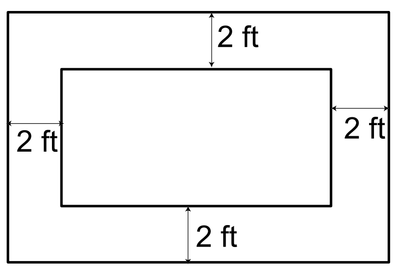


Draw the graph of any antideriviative of $f(x)$:

Draw a graph of the function $F(x) = \int_2^x f(t)\ dt$:

66. A farmer wishes to build a fenced coral with a feeding trough through the center. Fencing material costs \$45 per foot while it costs \$153 per foot of length to build the feeding trough. If the coral must have an area of 8100 ft.^{2} on each side of the trough, what is the lowest possible total cost? (The width of the feeding does not matter to the cost.)

67. A landscaper is asked to design and build a fenced-in rectangular flower bed. The flower bed is to be made up of two different varieties of flowers with one forming a 2 foot border about the other.



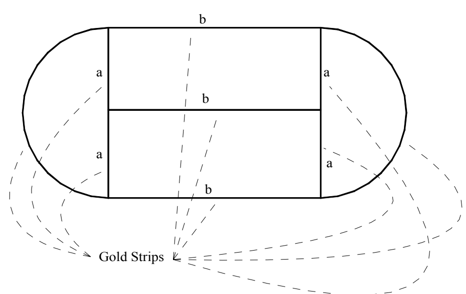
The garden's owner already has 100 ft^2 of the flower she wants to use on the outside border. Since these flowers are alreadt paid for, the landscaper's only profit will come what he makes from selling the fence and the inner flowers. The landscaper should choose the dimensions of the bed to maximize the cost to the owner. If the fencing costs \$25 for each foot of length and the second flower variety costs \$38 per ft^2, what is the maximum possible cost for such a garden?

68. The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$. The lateral surface area is $S = \pi r\sqrt{r^2 + h^2}$. What is the minimal lateral surface of a cone with volume 100 in^3? Strong hint: Minimize the square of the lateral surface area to make the algebra easier, but don't forget to answer the question asked.

69. Consider the family of functions $f(t) = A\,e^t + B\,t\,e^t$ where A and B are parameters. Find exact values of A and B so that $f(0) = 2$ and $f'(0) = 1$

70. A rectangular box with a square top and bottom has a fixed volume V. It must be constructed from three different kinds of material. The material used for the four sides costs \$1.28 per square foot; the bottom material costs \$3.39 per square foot, and the material for the top costs \$1.61 per square foot. Find the minimum cost for such a box. (Your answer should depend on V.)

71. An architect wants to feature a 100 ft^2 window on the corporate headquaters of a large company. The window will be in four sections, two semicircles and two rectangles. The glass sections will be separated by gold strips and the window's outside border will also be lined with the same strips. When assembled the window will look like the diagram below.



What dimensions, a and b, of the window will require the shortest total length of gold strips?

72. Local artist Daniel Bernoulli has never sold any of his sculptures. Perhaps this is because all his sculptures are right circular cones where the sum of the length of the radius and the height is always exactly 3 feet. Finally a local manufacturer commissions D. B. to build a cone with the largest possible volume (keeping the three foot rule) so that he can cover the lateral surface with the rust proofing he makes.
What is the lateral surface area of this cone with the largest volume?
A right circular cone of radius r and height h has volume $V = \frac{1}{3}\pi r^2 h$ and lateral surface area $A = \pi r\sqrt{r^2 + h^2}$.

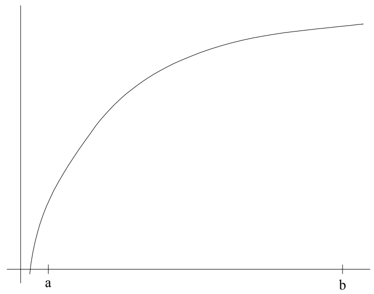
73. Consider the function $y = f(x)$ near the point $(-2, 3)$ given implicitly by

$$y^3 + 2x^2y + x^2 - 7x = 69$$

a) Find $\frac{dy}{dx} = f'(-2)$
b) Find the equation of the tangent at $(-2, 3)$.
c) Does this tangent line hit the curve $y^3 + 2x^2y + x^2 - 7x = 69$ in a point other than the point of tangency?

74. Find <u>exact</u> values for the constants a and b so that $f(\frac{1}{3}) = 1$ and $f(x)$ has a maximum at $x = \frac{1}{3}$, where $f(x) = a\,x\,e^{bx}$

75. Mark the following quantities on the graph of $f(x)$ given below. Explain each of your answers:



a) A length representing $f(b) - f(a)$
b) A slope representing $\frac{f(b)-f(a)}{b-a}$
c) An area representing $F(b) - F(a)$, where $F'(x) = f(x)$.
d) A length representing (roughly) $\frac{F(b)-F(a)}{b-a}$, where $F'(x) = f(x)$.

76. For any pair of functions $y = f(x)$ and $y = g(x)$, consider a third function $h(x) = f(g(x))$. No matter what pair you have, it is not possible that all three of the functions

$$f(x), \quad g(x), \text{ and } h(x) = f(g(x))$$

are decreasing for all values of x. Using derivatives, explain why this cannot happen.

77. Consider the family of functions

$$f(t) = A \sin 3t + A \cos 3t + B \sin 8t + B \cos 8t$$

where A and B are parameters.Find exact values of A and B so that

$$f(0) = 2 \text{ and } f'(0) = 1$$

78. Let $f(x)$ and $g(x)$ be functions. Suppose both functions are differentiable for $0 \leq x \leq 5$. Let

$f(1) = 4$; $f'(1) = -2$; $g(1) = 2$; $g'(1) = 1$
$f(2) = 2$; $f'(2) = -3$; $g(4) = 3$; $g'(4) = -2$

Consider the functions:

$$p(x) = f(x) \cdot g(x); \quad q(x) = \frac{f(x)}{g(x)} \quad \text{and} \quad h(x) = g(f(x))$$

Compute:
a) $h(1) =$ b) $p(1) =$ c) $q(1) =$ d) $h'(1) =$ e) $p'(1) =$ f) $q'(1) =$

79. If you invest P dollars in a bank account at an annual interest rate of r %, after t years you will have B dollars, where

$$B = P\left(1 + \frac{r}{100}\right)^t.$$

a) Find $\frac{dB}{dt}$, assuming P and r are constant. In terms of money what does $\frac{dB}{dt}$ represent?
b) Find $\frac{dB}{dr}$, assuming P and t are constant. In terms of money what does $\frac{dB}{dr}$ represent?

80. A bottle of water is placed in a refrigerator. The temperature of the water at time t (measured in Fahrenheit) is given by $T(t) = 40 + 30\,e^{-2t}$. If possible, answer each of the following questions with a full justification of your answer.
a) Is the temperature of the water increasing or decreasing?
b) Is the rate at which the temperature of the water is changing increasing or decreasing?
c) How cold is it inside the refrigerator?

81. Torricelli's Theorem says that if there is a hole in a container of liquid h feet below the surface of the liquid, then the liquid flows out at a rate given by

$$r(h) = \sqrt{2\,g\,h}$$

where $g = 32$ ft./sec^2 , the acceleration due to gravity. Find a linear function, $l(h) = m\,h + b$ that gives a good approximation of this rate for holes approximately 25 ft. below the surface of the water.

82. Find $\frac{dy}{dx}$ where $\frac{x^2 y}{y^2+4} = 5$

83. Find the equation of the line tangent to the curve $e^{2x} + e^{2y} = e^4 + e^2$ at the point $(2, 1)$.

84. Find a) $\int \frac{x}{x^2-3}\ dx$ b) $\int \frac{x^2-3}{x}\ dx$

85. Find the exact area between the two parabolas $y = 3x^2 - 2x + 5$ and $y = 2x^2 + x + 9$. Draw a figure illustrating the area you calculate.

86.Consider the family of functions $f(x) = e^x + k$. What value of k gives the curve that is tangent to the line $y = 4\,x + 5$? An exact answer will earn full credit, an approximate answer with a full explanation may earn partial credit.

87. For what value(s) of the parameter k does $f(x) = x^3 - kx^2 + kx + k$ have an inflection point at $x = 5$.

88. A rocket is designed so that for 10 seconds its acceleration increases linearly from zero acceleration to c m/sec^2 where $c > 0$. Draw and label graphs that illustrate the acceleration, the velocity, and the distance from the starting point as a function of time. Label at least one point (other than the origin) on each graph. (If you do not clearly label which graph is which, you may lose credit.)

89. A water tank has the form of a right circular cylinder. The tank is 30 ft in height with a radius of 8 ft. A hose is attached ta a drain in the tank and the valve is opened causing water begin to flow out of the tank. At one point the water in the tank drops from 3 ft deep to 2 ft, 9 inches in one miunute, estimate the rate at which the water is flowing through the hose.

90. According to a book of mathematics tables,

$$\int_0^{\pi} \ln(5 + 4\cos(x))\, dx = 2\pi \ln(2)$$

a) Use the formula and a graph of the function $f(x) = \ln(5 + 4\cos(x))$ to find the exact value of: (Explain your answer)

$$\int_{-\pi}^{\pi} \ln(5 + 4\cos(x))\, dx$$

b) Use the formula and a graph of the function $f(x) = \ln(5 + 4\cos(x))$ to find the exact value of: (Explain your answer)

$$\int_0^{3\pi} \ln(5 + 4\cos(x))\, dx$$

c) Use the formula and substitution to find the exact value of: (Explain your answer)

$$\int_0^{\frac{\pi}{2}} \ln(5 + 4\cos(2x))\, dx$$

91. Consider the family of parabolas $y = (a + 3) + 6\,a\,x - a^2\,x^2$ with $a \neq 0$.
a) Explain how you know that each parabola in the family has a unique point where the y-coordinate is a maximum.
b) Find this maximum y value as a function of the parameter a.

92. Which of the following phrases best describes the functions below: *You **absolutely must** explain your answer to receive credit. There will be no credit given for a correct choice of phrase without an explanation.*

a) $f(x) = x^4 + 4x$

Concave up   Concave down   A mixture of concave up and concave down

b) $f(x) = \cos x$

Concave up   Concave down   A mixture of concave up and concave down

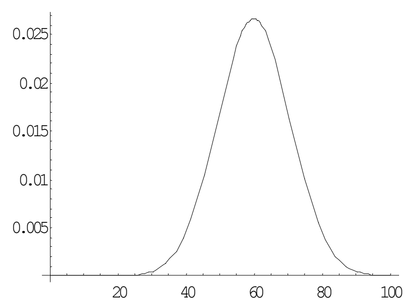
c) The smooth function illustrated by the table of values below.

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	1.1	1.5	1.7	1.8	1.85	1.851

(The values in the table are representative of the properties of the function.)

Concave up   Concave down   A mixture of concave up and concave down

d) Where $y = f(x)$ is graphed below



Concave up   Concave down   A mixture of concave up and concave down

93. Sketch a graph of the function

$$f(x) = \frac{(x^2+2)^7}{(x^2+1)^6}$$

Give exact values for the location of all local maximims and local minimums (Points of inflection are not required.)

94. Consider the curve parametrized by $x(t) = t^2 + 2t - 6$ and $y(t) = t^3 - 2t + 3$
a) Find the point on this curve at $t = 2$. b) Find the slope of the tangent line to the curve at $t = 2$

95. What is the exact point on the curve parametrized by $x(t) = 7t^2 + 5t - 6$ and $y(t) = 8t^3 - 2t + 3$ that is furthest to the left on the plane?

96. What is the area of the largest rectangle with one side on the x-axis that can be inscribed under the graph of $y = 25 - x^2$. You must show your work for credit.

97. Which of the following parametric curves are lines, which are circles, and which are neither.

a) $x(t) = \cos(t)$
$y(t) = \sin(t)$
Line Circle Neither

b) $x(t) = 15t + 3$
$y(t) = 28t + 16$
Line Circle Neither

c) $x(t) = \cos(125t)$
$y(t) = \sin(125t)$
Line Circle Neither

d) $x(t) = \cos(\sinh(t))$
$y(t) = \sin(\sinh(t))$
Line Circle Neither

e) $x(t) = \cosh(t)$
$y(t) = \sinh(t)$
Line Circle Neither

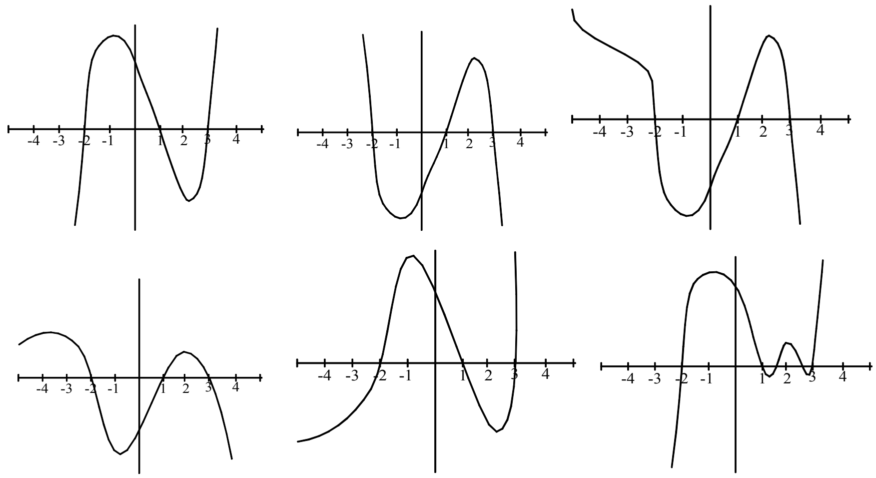
f) $x(t) = t^5 - 11t^4 + 16t^3 + 18t^2 + 4t - 5$
$y(t) = t^5 - 11t^4 + 16t^3 + 18t^2 + 4t - 5$
Line Circle Neither

98. Consider the lines parametrized by

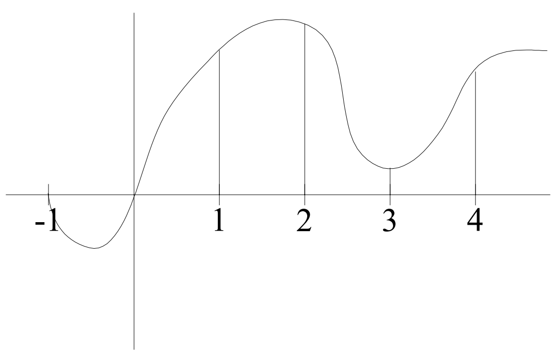
$$x(t) = 3t - 7 \quad \text{and} \quad x(t) = 5t + 6$$
$$y(t) = 4 - 9t \qquad\qquad y(t) = ct + 8$$

a) For what value of, if any, c are these two lines parallel?
b) For what value of c, if any, do these lines intersect at $(5, -32)$?
c) For what value of c, if any, are these lines the same?

99. Suppose you have function $y = f(x)$ with the following properties:
a) $f(x)$ is continuous and has a first and second derivative at every real number x.
b) $x = -2, 1,$ and 3 are solutions to $f(x) = 0$ and they are the only ones.
c) $x = -1$ and 2 are solutions to $f'(x) = 0$ and they are the only ones.
d) $x = 1$ is a solution to $f''(x) = 0$ and it is the only one.
Which of the following could be the graph of $y = f(x)$? (There might be more than one.)



100. Consider the function $f(x)$ whose graph is



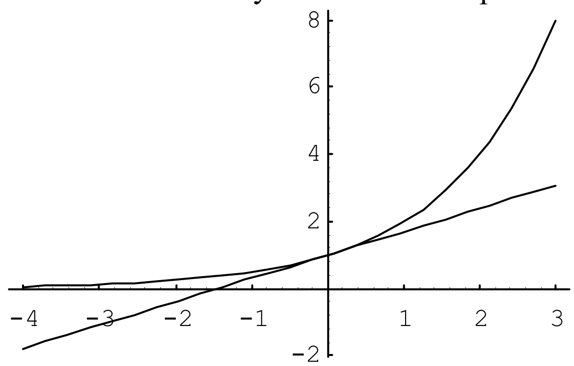
Order the following definite integrals from smallest to largest.

$$A = \int_{-1}^{0} f(x)\,dx \qquad B = \int_{-1}^{1} f(x)\,dx \qquad C = \int_{0}^{2} f(x)\,dx$$
$$D = \int_{2}^{2} f(x)\,dx \qquad E = \int_{0}^{3} f(x)\,dx \qquad F = \int_{2}^{4} f(x)\,dx$$

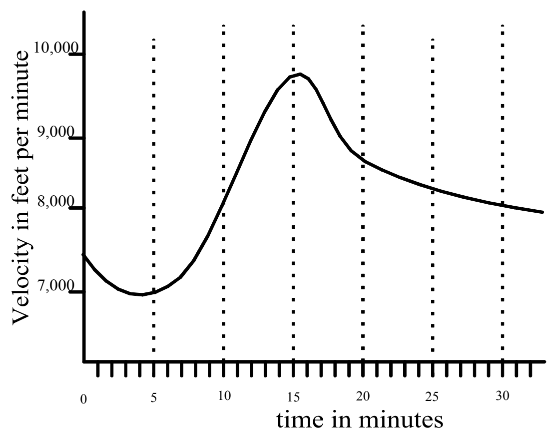
101. Evaluate exactly. Points may be deducted if work not shown.
a) $\int_{-1}^{1} e^{2x}\,dx$ b) $\int_{0}^{\frac{\pi}{2}} \sin(x)\,dx$ c) $\int_{0}^{1} (t^3 - t^2 - t - 1)\,dt$

102. You are asked to build a rectangular box, top, bottom and all four sides. It must have a bottom that is twice as long as it is wide. Its volume must be 2,000 cm^3. What dimensions should the box have to minimize the area of the six sides?

103. Below there is a graph of the equation $y = 2^x$ and the line tangent to the graph at the point (0,1). Just as the picture illustrates, the tangent line crosses the x-axis. Find the x coordinate of the point where the tangent crosses the axis and give your answer as an *exact* real number. Explain your answer. (You can get partial credit for a good decimal approximation of this coordinate if you include an explanation of how you found it..)



104. A helicopter flys along a north-south straight line directly over a radar site (at a fixed geographical point.) Throughout the flight, its velocity is recorded. Velocity toward the north is recorded as positive and velocity toward the south is recorded as negative. The velocity is graphed below



a) Does the helicopter reverse its direction during the recorded flight? If so, at what approximate time or times?

b) Does the helicopter change its velocity during the recorded flight? If so, approximately when is it flying fastest and when is it flying slowest?

c) Does the helicopter change its acceleration during the recorded flight? If so, approximately when is it accelerating the most?

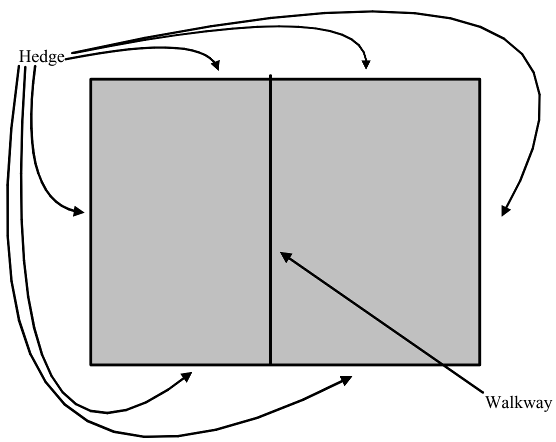
d) Does the helicopter ever decelerate during the recorded flight? If so, approximately when is it decelerating the most?

105. a)Find the exact value of $\int_0^1 (x^2 - 2x)\, dx$ b) Find $\int \sin x \cos^3 x \, dx$

106. A model vehicle is run off an experimental motor through a transmission with two gears. When it is run from a dead stop, it has a constant acceleration of 12 ft/sec^2 for the

first 5 seconds. At this points the gears are changed (instantaneously) and the acceleration changes to 10 ft/sec^2 for the next 4 seconds. After that, the motor is turned off. This problem is about these first 9 seconds.
a) Draw a graph of the velocity as a function of time. Be sure to label the y-axis.
b) If you were to draw a graph of distance traveled as a function of time (you do not need to actually do this), would it be continuous when the time is 5 seconds, ie. when the gears are changed? You must explain your answer. A simple 'yes' or 'no' will receive no credit.

107. An architect designs a rectangular garden. The garden is to be surrounded on all sides by a hedge, and a trellised walkway runs between one pair of opposite sides. The hedge costs \$98 per linear foot of length, the walkways costs 127\$ per linear foot. The garden must have a total area of 1000 ft^2. What dimension should the garden have to minimize the cost?



108. A gardener wants to plant a rectangular plot with hedges on three of the sides and an ornamental fence on the final side. The hedge he has in mind costs \$45 per linear foot to plant and the fence will cost \$125 per foot to build. What is the minimum cost of a plot that is 1000 ft^2 in area?

109. The absolute value function $f(x) = |x|$ is differentiable at every $x \neq 0$. In fact

$$f'(x) = \frac{x}{|x|}$$

(Hint: use the chain rule when necessary.)
a) Find the derivative of $g(x) = |x^3 - 2x|$ at the points where it exists.
b) Find the derivative of $h(x) = |x|^2$ at the points where it exists.

110. What is the area of the largest rectangle with one side on the x-axis that can be inscribed under the graph of $y = 25 - x^2$. You must show your work for credit.

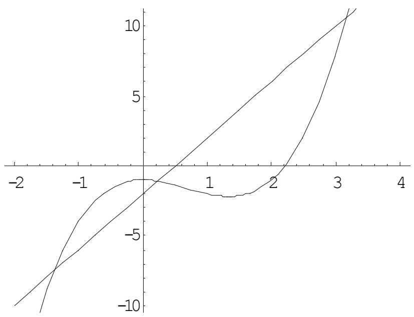
111 For what values of the parameters a and b does the function

$$y = a\sinh(2x) + b\cosh(2x)$$

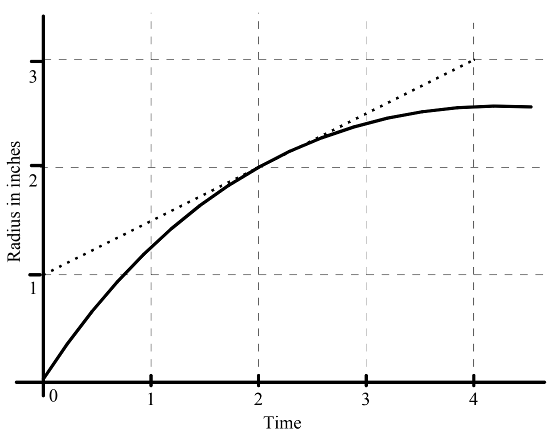
satisify

$$\frac{d^2y}{dx^2} = 16\sinh(2x) + 24\cosh(2x)?$$

112. The figure below shows the curve $y = x^3 - 2x^2 - 1$ and the line $y = 4x - 2$. It appears from the graph that there are two lines tangent to the curve $y = x^3 - 2x^2 - 1$ and parallel to the line $y = 4x - 2$. Show that this is true by finding the exact equation of each tangent line.



113. A spherical balloon is inflated so that the radius of the balloon at time t seconds is given by the graph: Radius in inches = r; Time = t.



The volume of a sphere of radius r is given by V $= \frac{4}{3}\pi r^3$. What is the **rate of change** of the volume, $\frac{dV}{dt}$, of this balloon at time $t = 2$?

114. Match the function with its derivative

____ $f(x) = 2^{\sin(x)}$

____ $f(x) = e^{2\sin(x)}$

____ $f(x) = e^{\ln(\sin(x))}$

____ $f(x) = (\sin(x))^{2e}$

____ $f(x) = e^{\sin(2)}$

a) $\frac{dy}{dx} = \frac{2e}{\sin(x)}(\sin(x))^{2e}\cos(x)$

b) $\frac{dy}{dx} = \cos(x)$

c) $\frac{dy}{dx} = 2\cos(x)\, e^{2\sin(x)}$

d) $\frac{dy}{dx} = 0$

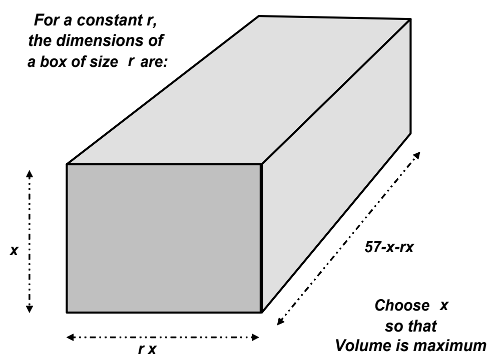
e) $\frac{dy}{dx} = \ln(2)\cos(x)\, 2^{\sin(x)}$

115. On the moon the acceleration due to gravity is 5.3 ft/sec^2. Suppose a rock on the moon falls from a height of 100 ft starting at rest at time zero. a) Find a formula for the velocity of the rock at time t.
b) Find a formula for the height of the rock at time t. (You must show **all** your work for full credit.)

116. Find the equation of the tangent to the curve $y^2 + x\,y + x^2 = 3a^2$ at the point (a, a)

117. A pile of crushed rock is increasing in volume at a rate of $\frac{dV}{dt} = 23$ ft^3/min. The pile has the rough form of a right circular cone. At one point, the pile has a height of $h =$ 4 ft and radius of $r = 6.5$ ft which is expanding at a rate of $\frac{dr}{dt} = 0.25$ ft/min. How fast is the height of the cone growing at that moment. (The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

118. A shipping company has decided to limit the packages it accepts for overnight delivery to rectangular boxes with the sum of the three dimensions, length, width and height less than 57 inches. A box manufacturer wants to make several different boxes that fit these restrictions, but have the largest possible volume. The owner suggests that the different boxes be distinguished by the ratio of two of the sides. For example a size 1 box would have two equal sides, a size 2 box would have one side twice another side, and in general a size r box would have one side r times the other. But one employee points out, if this is the way they produce the boxes, then all their boxes will have one side with length 19 inches. Is the employee right or wrong? Justify your answer.

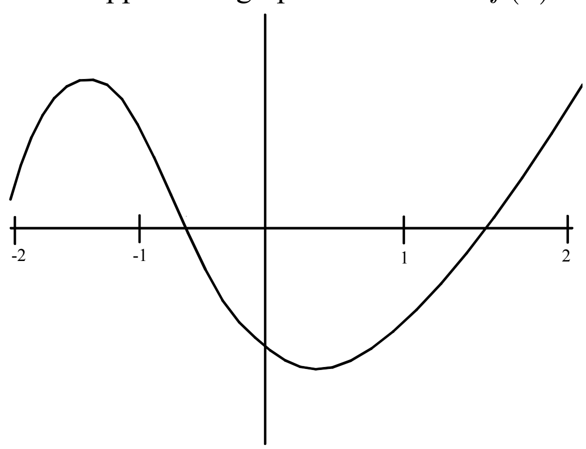


119. Differentiating the equation $x^2 + xy + y^2 = 19$ implicitly gives $\frac{dy}{dx} = \frac{-2x-y}{x+2y}$.
a) Use this to find the equation of the tangent line at the point $(2, 3)$.
b) Use the second derivative to determine if this tangent line is above or below the curve near the point of tangency.

120. Let $f(x)$ be a function that is **positive** and **decreasing** for every real number x. Fill in the "answer" column following chart with:
"Increasing": if the function $g(x)$ is increasing for all real values of x.
"Decreasing": if the function $g(x)$ is decreasing for all real values of x.
"Some of each": if the function $g(x)$ is increasing for some real x and decreasing for others.
"Cannot tell": if there is not enough information to determine that one of the above is true about $g(x)$.
An explanation for each answer is required. (Hint; Use derivatives.)

Function	Answer	Explanation
$g(x) = f(x^2)$		
$g(x) = (f(x))^2$		
$g(x) = f(x+1)$		
$g(x) = f(x) + x$		
$g(x) = -f(x)$		

121. Suppose the graph of a function $f(x)$ is given in the figure below.



a) Order the following numbers from smallest to largest

$$f(-2);\ f(-1);\ f(0);\ f(1);\ f(2)$$

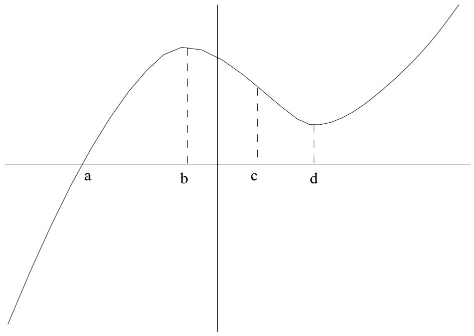
b) Order the following numbers from smallest to largest.

$$f'(-2);\ f'(-1);\ f'(0);\ f'(1);\ f'(2)$$

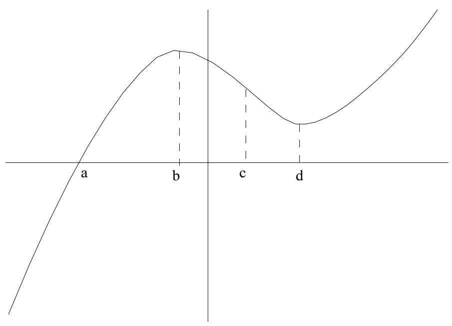
c) Order the following numbers from smallest to largest.

$$\int_{-2}^{-1} f(x)\,dx,\quad \int_{-1}^{1} f(x)\,dx,\quad \int_{1}^{2} f(x)\,dx$$

122. a) Sketch two functions $y = f(x)$ in the family of functions where $f'(x)$ has the graph.



b) Sketch a graph of the derivative of $y = f(x)$ where $f(x)$ is the function graphed below.



123. Let $f(x)$ be a function. Suppose its values and the values of its derivative are given by

x	$f(x)$	$f'(x)$
1.0	1.414	1.061
1.2	1.625	1.046
1.4	1.833	1.037
1.6	2.040	1.030
1.8	2.245	1.025
2.0	2.450	1.021

Use this table and the chain rule to develop tables for the following functions:

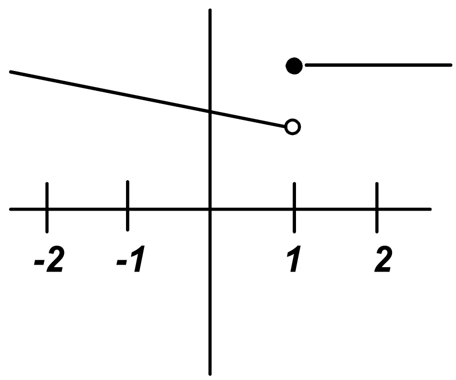
$g(x) = f(2x)$

x	$g(x)$	$g'(x)$

$h(x) = (f(x))^2$

x	$h(x)$	$h'(x)$

124. In the following graphs, **Circle** all the phrases that are correct.



continuous at x=1

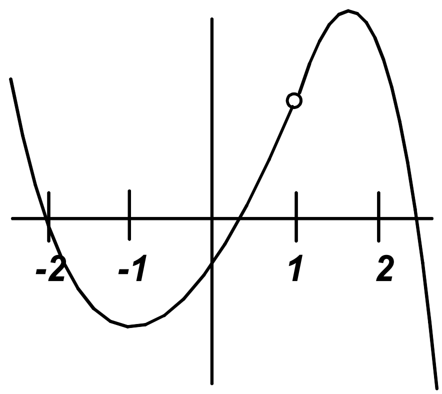
differentiable at x=1

Limit as x approaches 1 exists

continuous at x= -1

differentiable at x= -1

Limit as x approaches -1 exists



continuous at x=1

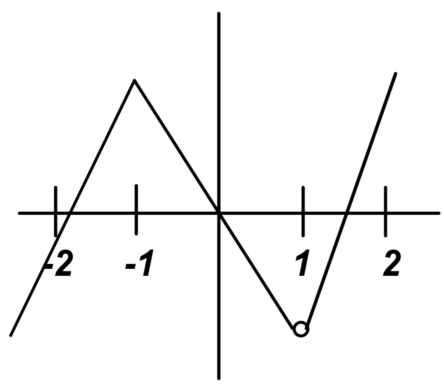
differentiable at x=1

Limit as x approaches 1 exists

continuous at x= -1

differentiable at x= -1

Limit as x approaches -1 exists



continuous at x=1

differentiable at x=1

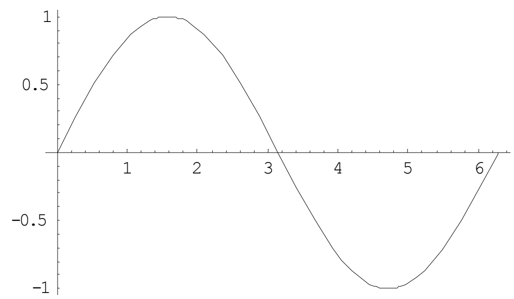
Limit as x approaches 1 exists

continuous at x= -1

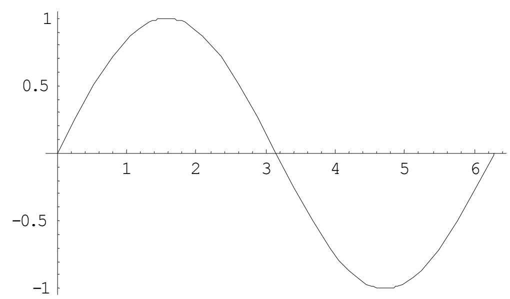
differentiable at x= -1

Limit as x approaches -1 exists

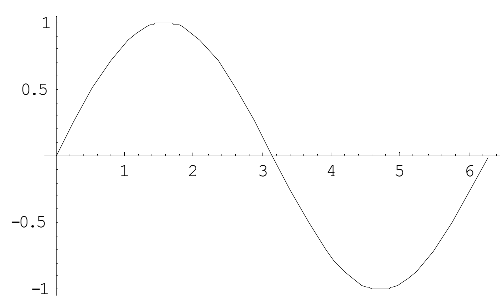
125. The graph of the function $f(x) = \sin(x)$ is given next to each of the following integrals. Shade in a region on the graph which has an area given by the integral. (You will be given a piece of scratch paper with copies of the graph on it to prepare your answer. ***Only the answers given below will be graded,*** and any ambiguous drawings including incomplete erasures or crossed out work may be marked as wrong.)



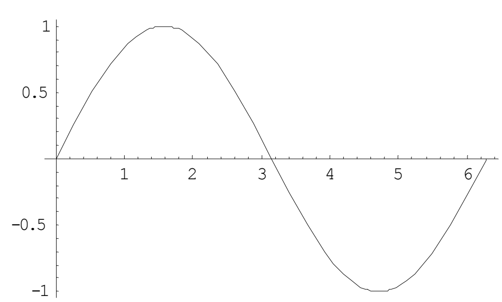
$\int_0^{\pi} \sin(x)\,dx$



$\int_0^{2\pi} |\sin(x)|\,dx$

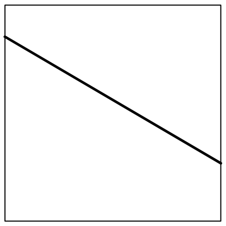


$\int_0^{\frac{3\pi}{2}} \sin(x)\,dx$

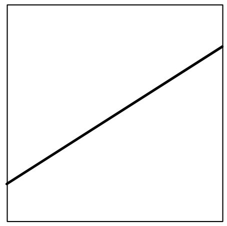


$\int_0^{2\pi} 1 - \sin(x)\,dx$

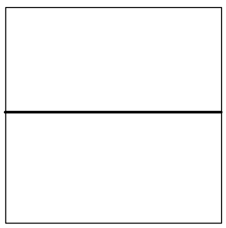
126. Consider the function $f(x) = x^{17} + 14x^{15} + 129$. If you graph this function on your calculator, you will get different views depending on the window you choose. Some may be possible by zooming in very close to a point, others by zooming out.
Which of the following views are actually possible?



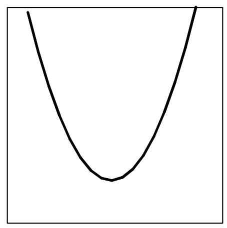
YES NO



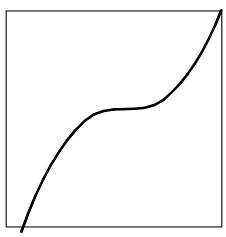
YES NO



YES NO

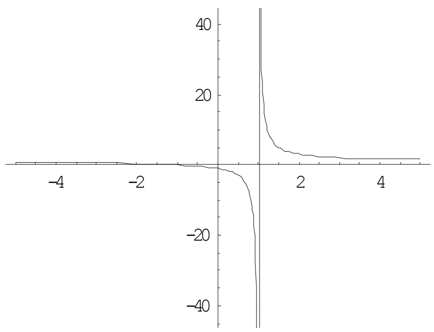


YES NO

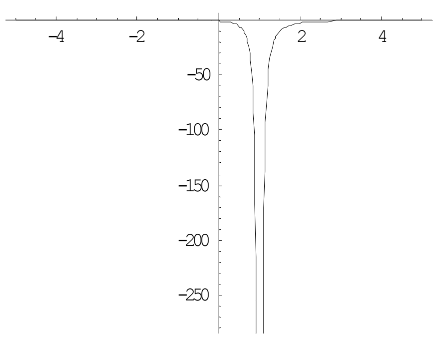


YES NO

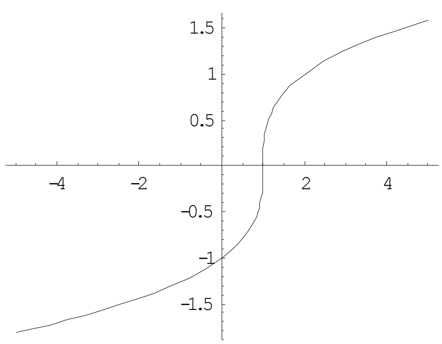
127. Match the graph of the function with the graph of its derivative



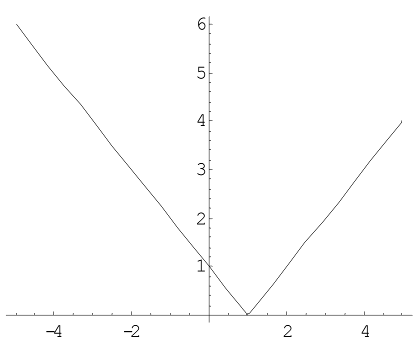
Answer____________________



Answer____________________

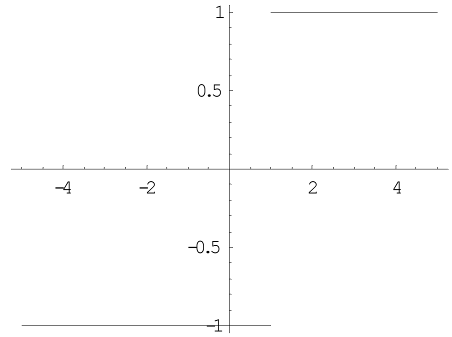


Answer____________________

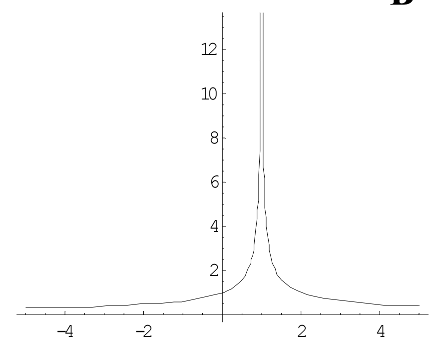


Answer____________________

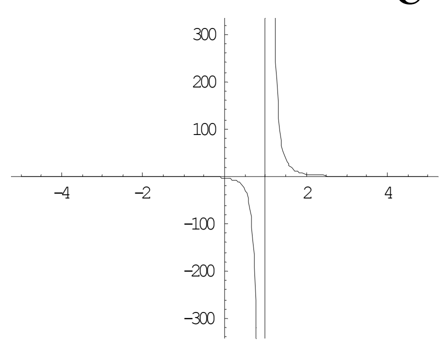
A



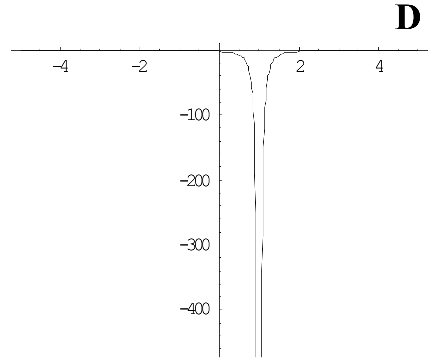
B



C



D



128. Find the derivatives of the following functions:

a) $s(t) = t^2 + \frac{1}{t^2}$ b) $f(x) = e^{\sin(x)}$ c) $g(x) = \frac{1+2x}{2x+3}$

d) $s(t) = \arctan(2t)$ e) $r(t) = t\cos(t)$

129. A function $f(x)$ is approximated in the table below .

x	2.00	2.15	2.30	2.45	2.60	2.75	2.90
$f(x)$	2.32	2.33	2.35	2.39	2.47	2.63	2.95

(The values in the table are representative of the properties of the function.)

Use this table to approximate the values in the following table of the derviative:

x	2.075	2.225	2.375	2.525	2.675	2.825
$f'(x)$						

130. Find an exact value for the constant b so that $f(x)$ has a maximum at $x = 3$, where

$$f(x) = (x - b)\, e^{bx}$$

131. What is the exact value of smallest possible slope that a line tangent to $f(x) = e^{x^2-4}$ can have at an x value satisfying $1 \leq x \leq 4$. (You must justify your answer!)

132. Evaluate the following exactly::

a. $\int_0^3 x^3\, dx$ b. $\int_0^{\pi} \sin(x)\, dx$ c. $\int_{-3}^{-1} \frac{5-x}{x}\, dx$

133. Suppose $f(x)$ and $g(x)$ are functions so that

$f(1) = 1$ and $g(1) = 6$ and $f'(1) = 8$ and $g'(1) = -2$

Find

a) $h'(1)$ where $h(x) = 2f(x) + 3g(x)$: $h'(1) =$

b) $k'(1)$ where $k(x) = f(x)g(x)$: $k'(1) =$

c) $l(1)$ where $l(x) = \frac{3f(x)}{2g(x)}$: $l'(1) =$

d) $m(1)$ where $m(x) = g(f(x))$: $m'(1) =$

d) $n(1)$ where $n(x) = (f(x))^2 + (g(x))^2$: $n'(1) =$

134. Consider the curve parametrized by

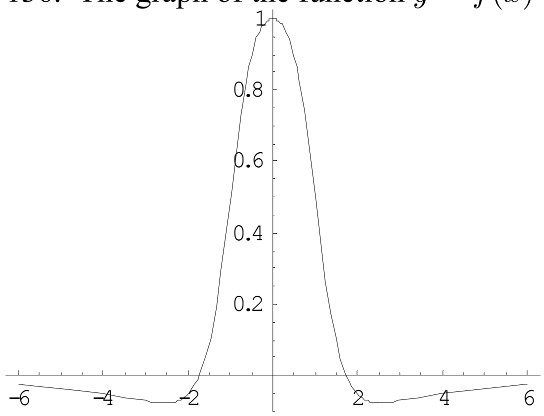
$$x(t) = 2\sin(3t)$$
$$y(t) = 3\sin(2t)$$

a. Does this curve go through the point $(3, 2)$?
b. Does this curve go through the point $(2, 3)$?
c. Does this curve go through the point $(2, 0)$?
d. What is the slope of the tangent to this curve at $t = \frac{\pi}{3}$?

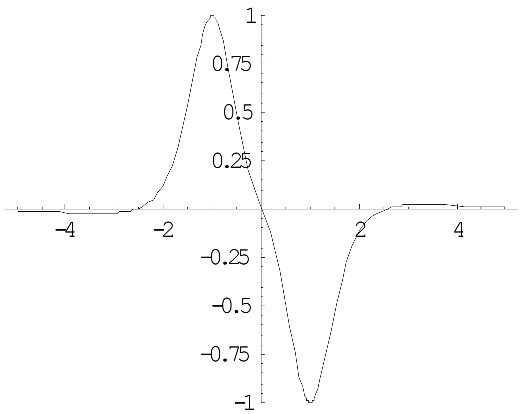
135. Find the following limits exactly:

a. $\lim_{x\to 0} \frac{\sin(7x)}{x}$ b. $\lim_{x\to \pi} \frac{\sin(7x)}{x}$ c. $\lim_{t\to\infty} \frac{3t^2+2t+1}{13t^2-8}$ d. $\lim_{y\to -\infty} e^{2y}$

136. The graph of the function $y = f(x)$ is



The graph of its derivative $f'(x)$ is



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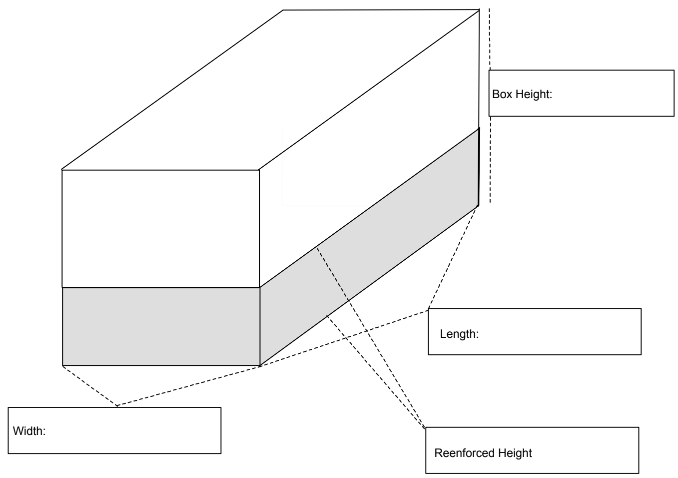
Sketch a graph of the function

$$g(x) = f(x)^2$$

Important points on the graph must be labeled.

137. A box manufacturer is designing a rectangular shipping container to hold ball bearings. Because of the weight of these, the bottom of the container must be made from his sturdiest material. This material costs $22.00 per square foot. Further the bottom 1/3 of every side must be made of this same material. The rest of the sides and the top can be made from material costs $5.00 per square foot Finally the bottom of the box should be twice as long as it is wide.

What are the dimensions of the least expensive box with a volume 16 ft^3 that meets these specifications? Place the dimensions on the diagram below.



For partial credit answer the following:
Using l for the length; w for the width, and h for the height. Give answers in these terms!:

f) What is the cost of the bottom?
g) What is the cost of the top?
h) What is the cost of bottom part of **one** long side?
i) What is the cost of top part of **one** long side?
j) What is the cost of bottom part of **one** short side?
k) What is the cost of top part of **one** short side?
l) What is the total cost of a complete box? (Include all 6 sides.)
Solve the problem.