# MATH 129, HW 8
# DUE WE 04/17/2002

**Question 1: (5pts) Do Marcus #33 p. 152 and apply it to your 3 favourite values of $m$.**

**Question 2: (12pts) Do Marcus p155-157:**

(1) **(3pts)** #43.
(2) **(2pts)** #45
(3) **(4pts)** #46
(4) **(3pts)** #48.

**Question 3: (18pts).** Let $K \subset L$ be two number fields, $R \subset K$ and $S \subset L$ their number rings.

Define $i : I(R) \to I(S)$ by $i(a) = Sa$.

(1) Define $\pi : I(S) \to I(R)$ by $\pi(b) = K \cap b$.
 (a) **(1pt)** Show that $\forall a \in I(R)$, $\pi i(a) = a$.
 (b) **(1pt)** Is $\pi$ a group homomorphism?
(2) For a prime ideal $Q \subset S$, define $N_{L/K}(Q) = P^{f(Q,P)}$, where $P$ is the unique prime ideal of $R$ below $Q$. This extends by linearity to a group homomorphism $N_{L/K} : I(S) \to I(R)$.
 (a) **(6pts)** Do Marcus #15 p.85.
 (b) **(1pt)** Show that $N_{L/K} \circ i(a) = a^{[L:K]}$.
 (c) **(2pts)** Do Marcus #16 p.86.
 (d) **(1pt)** Show that if $[L : K]$ is prime to $\#\mathrm{Cl}(K)$, then $\#\mathrm{Cl}(K) \mid \#\mathrm{Cl}(L)$.
 (e) **(5pts)** Do Marcus #17 p.86.
(3) Suppose that $L/K$ is an abelian extension. Let $I(R, d)$ be the set of elements of $I(R)$ which are relatively prime to $d = \mathrm{disc}(L/K) \subset R$. For $P \nmid d$, let $\mathrm{Artin}(P) \in \mathrm{Gal}(L/K)$ be the Frobenius element corresponding to $P$. This extends by linearity to $\mathrm{Artin} : I(R, d) \to \mathrm{Gal}(L/K)$.
 **(1pt)** Show that $\mathrm{Im}(N_{L/K}) \cap I(R, d) \subset \mathrm{Ker}(\mathrm{Artin})$.

**Question 4: (6pts) Do Marcus #5 p.115.**