Math 594. Homework 4 (due February 5)

Book problems: In §4.5, read pp. 146–7 and then do Exercises 6,7, 13, 25. In §5.5, read pp. 184–6, then do Exercises 6–9.

1. Let G be a finite p-group.

(*i*) For every proper subgroup H in G, show that H is a proper subgroup of its normalizer.

(*ii*) Deduce that there exists a solvability series for G in which one of the terms in actually H.

(*iii*) Give an example of a non-trivial quotient situation G/H for which there does not exist a subgroup K in G with $K \to G/H$ an isomorphism (concretely, we can't "lift" G/H *as a group* back into G). Hint: think of cyclic groups.