

MATH 223, Linear Algebra
Fall, 2007

Assignment 4, due *in class* Friday October 5, 2007

1. Consider the vector space $V = P_5(\mathcal{R})$ of polynomials with real coefficients (in one variable t) of degree at most 5 (including the zero polynomial). Show that if $c \in \mathcal{R}$ is any real number, then the set

$$\{1, t - c, (t - c)^2, (t - c)^3, (t - c)^4, (t - c)^5\}$$

is a basis of V .

2. Let V be as in problem (1). Let W be the subset of V consisting of those polynomials $p(t)$ such that $p(0) = 0$, and let U be the subset of polynomials $p(t)$ that are *even*, i.e. such that $p(t) = p(-t)$.

(a) Show that U and W are subspaces of V .

(b) Compute $\dim V$, $\dim W$, $\dim U$, $\dim U \cap W$.

3. Do the vectors $v_1 = (1, 2, 0, 3, 5)$, $v_2 = (2, 0, 3, 1, 7)$, $v_3 = (3, 5, 1, 0, 2)$, $v_4 = (1, 1, 0, 9, 4)$, $v_5 = (8, 6, 7, 5, 3)$ form a basis of \mathcal{R}^5 ?

4. Let A and B be matrices, and suppose that the product AB is defined. If the column space of B is contained in the nullspace of A , show that $AB = 0$ (i.e. the product AB is equal to the zero matrix).

5. Let $V = \mathcal{C}$ be the complex numbers, considered as a vector space over the real numbers \mathcal{R} . It has dimension 2, and the vectors $e_1 = 1$, $e_2 = i$ form a basis for \mathcal{C} over \mathcal{R} . Consider the mapping

$$m_i : \mathcal{C} \rightarrow \mathcal{C}$$

defined by

$$m_i(z) = i \cdot z,$$

(where the multiplication $i \cdot z$ is as complex numbers). Show that m_i is a linear map and determine its matrix relative to the ordered basis $\{e_1, e_2\}$ of \mathcal{C} given above.

6. Let $V = P_3(\mathcal{R})$ be the real vector space of polynomials with real coefficients (in one variable t) with degree at most 3. Define a mapping

$$f : V \rightarrow V$$

by

$$f(p(t)) = t^3 p\left(\frac{1}{t}\right).$$

(a) Show that f is a linear map.

(b) Find the matrix of f relative to the ordered basis $\{e_1, e_2, e_3, e_4\}$ of V given by $e_1 = 1$, $e_2 = t$, $e_3 = t^2$, $e_4 = t^3$.

(c) Find a basis for the kernel of the linear mapping

$$g : V \rightarrow V$$

given by

$$g(p(t)) = f(p(t)) - p(t).$$