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MATHEMATICS 223, FALL 2005
LINEAR ALGEBRA
MIDTERM EXAMINATION, WEDNESDAY OCTOBER 19, 2005

Instructions:

1. There are six problems on the opposite side of this sheet. Do all six, in the booklets provided. They are of equal weight.
2. Books, notes, and calculators are not permitted.
3. Good luck!

(Instructions on the opposite side.)

1. (a) Express the following matrix A (over the complex numbers) as a product of elementary matrices. $A = \begin{pmatrix} 1+i & -2+2i \\ 3i & -4 \end{pmatrix}$.

   (b) For the above A, find A^{-1}.

   (c) Solve the system of linear equations $\begin{array}{ccccc} (1+i)x_1 & + & (-2+2i)x_2 & = & -1+3i \\ 3ix_1 & + & -4x_2 & = & i \end{array}$.

2. Let $V = M_2(\mathcal{R})$ be the vector space of 3×3 matrices over the real numbers. For each of the following subsets of V, decide whether or not it is a subspace of V. Justify your answers briefly.

   (a) $S_1 = \{A \in V : A^T = A \text{ and } A \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = \vec{0}\}$.

   (b) $S_3 = \{A \in V : A\vec{v} = \vec{v} \text{ for some nonzero vector } \vec{v}\}$.

3. Find a basis for each of the row space, column space and null space of the following matrix. What is its rank?

$$\begin{pmatrix} 1 & 0 & 3 & 1 & -4 \\ 2 & 1 & -1 & -1 & 3 \\ 6 & 2 & 4 & 0 & -2 \end{pmatrix}.$$

4. Let $W_1 = Span\{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}\}$ and $W_2 = Span\{\begin{pmatrix} 1 \\ 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 6 \\ 1 \end{pmatrix}\}$ be subspaces of $\mathcal{R}^4$.

   Find a basis for each of $W_1 + W_2$ and $W_1 \cap W_2$.

5. (a) Verify that $\mathcal{B} = (1, 1+t, (1+t)^2, (1+t)^3\}$ is a basis for $P_3(t)$, the vector space of polynomial over the reals with degree at most 3.

   (b) Find the coordinates of f with respect to $\mathcal{B}$ (in the given order), where $f(t) = t^3$.

6. Suppose that V is a vector space, and each of $B = \{\vec{v}_1, \ldots, \vec{v}_k\}$ and $C = \{\vec{w}_1, \ldots, \vec{w}_\ell\}$ is an independent subset of V, but $B \cup C$ is not independent.

   Letting $U = Span(B)$ and $W = Span(C)$, show that $U \cap W \neq \{\vec{0}\}$.