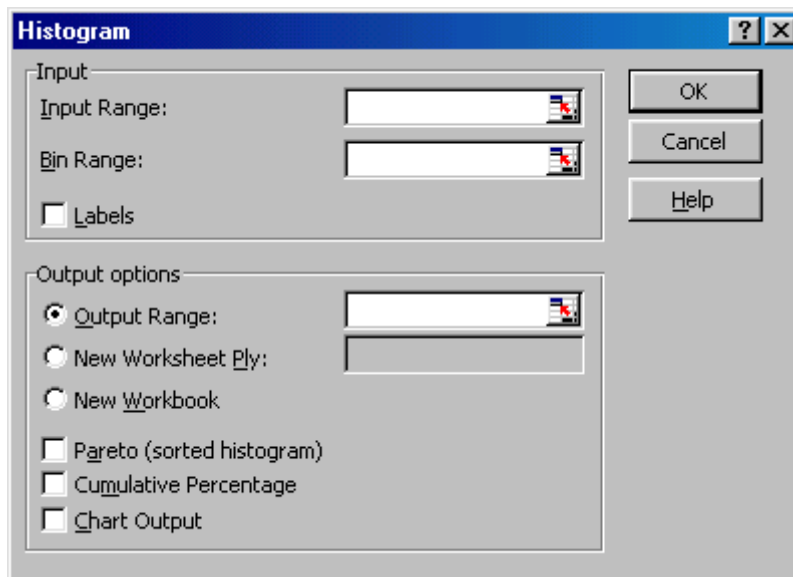


Business Mathematics I
TEST 2 STUDY GUIDE

1. Consider the data shown below.

	A	B	C	D	E
1	<i>Data</i>	<i>Bins</i>	<i>Bin</i>	<i>Frequency</i>	<i>Relative Frequency</i>
2	3.8	4	4		
3	10	6	6		
4	7	8	8		
5	9.2	10	10		
6	9	12	12		
7	5				
8	8.4				
9	9				
10	4.6				
11	6.2				
12	4.4				
13	11.4				

- (a) Fill in the ***Frequency*** and ***Relative Frequency*** columns, as they would be done by the ***Histogram*** and other functions in *Excel*. Do this manually, not with the help of *Excel*.
- (b) Fill in the information that would be needed to have *Excel* sort the data into the bins given in column B. (The output table should be displayed in columns C and D with labels in row 1.)



2. A sum of \$20,000 is deposited into a savings account that pays an annual rate of 4.9%.
 - (a) What is the future value of the account after five years if interest is compounded continuously?
 - (b) What is the effective annual yield on the account if interest is compounded continuously? Give your answer as a percentage rounded to two decimal places (i.e., x.xx%).
 - (c) What is the future value of the account after five years if interest is compounded quarterly?
 - (d) What is the effective annual yield on the account if interest is compounded quarterly? Give your answer as a percentage rounded to two decimal places (i.e., x.xx%).

3. What is the value of \$600 invested at 5%, compounded daily after a period of three years?
 - (A) \$694.58
 - (B) \$696.45
 - (C) \$696.88
 - (D) \$697.09
 - (E) \$697.10

4. Which investment will be worth more: \$1,000 invested at 5.9%, compounded monthly, or \$1,000 invested at 5.6%, compounded continuously?

5. Suppose that a \$50,000 payment is to be made eight years from now and that money can be invested at an annual rate of 6.5%.
 - (a) What is the present value of the payment if interest is compounded continuously?
 - (b) What is the present value of the payment if interest is compounded monthly?

6. How much money, to the nearest cent, should be deposited in an account paying 6.5% interest, compounded quarterly, in order to have a balance of \$64,000 in nine years?
 - (A) \$35,654.78
 - (B) \$35,822.83
 - (C) \$114,340.50
 - (D) \$114,879.42
 - (E) None of the above

7. A couple wants to set up a college account for their newborn child. They estimate that they will need \$15,000 dollars a year for each of four years, starting 18 years from now. How much would they need to invest now if money can be invested at an annual rate of 5.5%, compounded continuously?

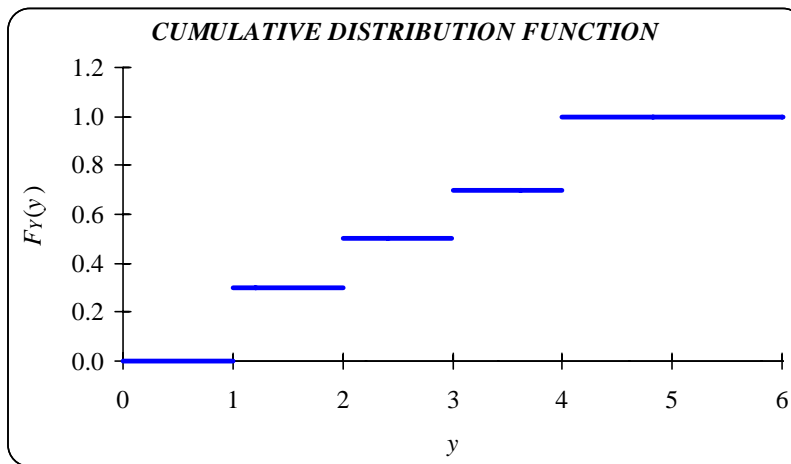
8. HF Credit Union offers a “high yield” savings account with an annual interest rate of 1.41%, compounded continuously. BA Bank offers a Money Market savings account with an annual interest rate of 1.44%, compounded monthly. Which account has the higher yield?
9. Find the annual rate that produces an effective annual yield of 5%, when compounded
- (a) Annually.
 - (b) Quarterly.
 - (c) Daily.
 - (d) Continuously.
10. You deposit \$50,000 in your bank account and plan to leave it there for five years. The current interest rate is 5.2%, compounded quarterly. Another bank offers an account in which interest would be compounded continuously. What interest rate would the second bank need to offer in order for the two accounts to be equally desirable? Give your answer as a percentage rounded to two decimal places (i.e., x.xx%).
11. Arrange the following from largest to smallest.
- (i) The effective annual yield on an investment of \$100,000 for a period of five years at an annual rate of 8%, compounded monthly.
 - (ii) The effective annual yield on an investment of \$500,000 for a period of four years at an annual rate of 6%, compounded continuously.
 - (iii) The effective annual yield on an investment of \$200,000 for a period of seven years at an annual rate of 9%, compounded quarterly.
- (A) (i), (ii), (iii)
 - (B) (i), (iii), (ii)
 - (C) (ii), (i), (iii)
 - (D) (iii), (i), (ii)
 - (E) (iii), (ii), (i)
12. How long does it take for an investment to double in value if it is invested at an annual rate of 8%, compounded
- (a) Annually?
 - (b) Semi-annually?
 - (c) Monthly?
 - (d) Continuously?

13. The city of Erehwon has installed wireless Internet within its city limits. The servers can handle up to 50,000 computers at a time without significant loss of performance. There are currently 35,000 computers that are regularly logged on to the wireless network. If the City Council expects the number of computers in Erehwon to grow exponentially at a continuous annual rate of 5.2%, how long (to the nearest month) will it be until they need to upgrade the wireless servers?
14. An investment of \$15,000 made in 2004 was worth \$20,000 three years later. What annual rate was earned on the investment if interest was compounded
- (a) Annually?
 - (b) Quarterly?
 - (c) Daily?
 - (d) Continuously?
15. Steve wants to buy a used car when he turns 16 in three years. He currently has \$4,200 and expects to pay \$5,000 for the car. At what annual rate would Steve need to invest his money if interest is compounded continuously? Give your answer as a percentage rounded to two decimal places (i.e., x.xx%).
16. Let X be the random variable giving the value of the S&P 500 Index on July 1, 2001. Is X finite or continuous? Is f_X a *p.m.f.* or a *p.d.f.*?
17. Let Y be the random variable giving the number of months in the year 2001 when the S&P 500 Index has a monthly high above 1,600. Is Y finite or continuous? Is f_Y a *p.m.f.* or a *p.d.f.*?
18. A random variable W has the following values and probabilities.

w	0	1	2	4	8	16
$P(W = w)$	0.1	0.1	0.2	0.2	0.3	0.1

- (a) Find $f_W(8)$.
- (b) Find $F_W(8)$.
- (c) Find $f_W(10)$.
- (d) Find $F_W(10)$.

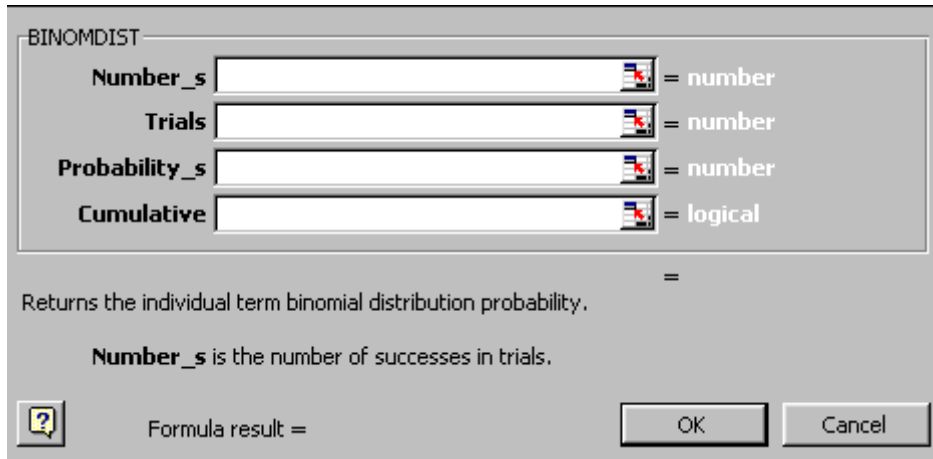
19. The graph of the *c.d.f.* of a finite random variable, Y , is given below.



- (a) Find $P(Y \leq 3)$.
 - (b) Find the possible values of Y and the corresponding values of the *p.m.f.*
 - (c) Draw a well-labeled graph of the *p.m.f.* of Y .
 - (d) Find μ_Y .
20. A courier van that runs between two of your company's facilities encounters five traffic signals along its route. The signals operate independently, with each having a probability of 0.55 that the van has to stop. Let X be the random variable giving the number of signals at which the van must stop on a randomly selected trip.
- (a) What kind of random variable is X ?
 - (b) Use *Excel* to generate a table containing all possible values of X and the corresponding values of f_X and F_X . Round your answers to 4 decimal places.
 - (c) On average, at how many of the signals can the courier van expect to stop?
21. Approximately 18% of all undergraduates at the University of Arizona are majoring in business. What is the probability that at least six undergraduates, in a random sample of size 10, are majoring in business? Round your answer to four decimal places.
- (A) 0.0004
 - (B) 0.0037
 - (C) 0.9963
 - (D) 0.9996
 - (E) None of the above

22. The probability that a patient recovers from a delicate heart operation is 0.9. Four patients are in the ICU recovering from surgery. Let X record the number of patients that survive.

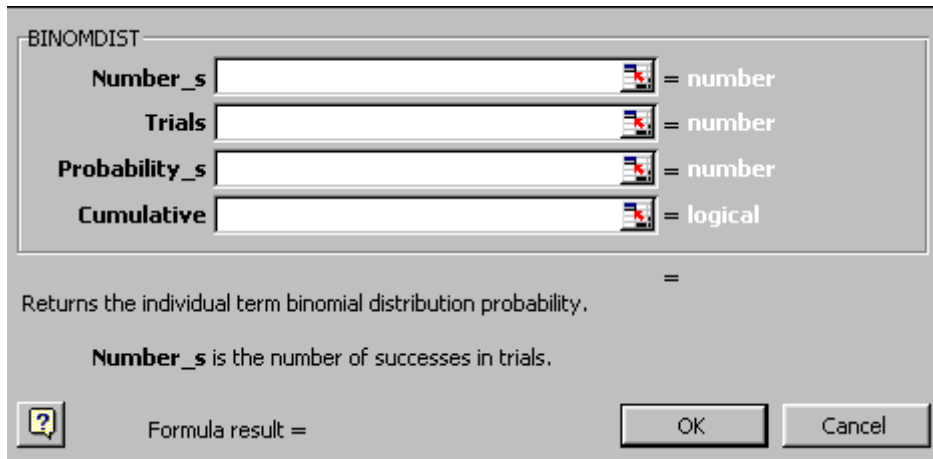
- (a) Fill in the information that would be needed to have *Excel* compute the probability that exactly one of the patients survives.



The image shows the Excel BINOMDIST dialog box. It has four input fields: "Number_s", "Trials", "Probability_s", and "Cumulative". Each field has a small icon to its right. To the right of each field is a label: "= number" for the first three and "= logical" for the last. Below the fields is a description: "Returns the individual term binomial distribution probability." and "Number_s is the number of successes in trials." At the bottom, there is a "Formula result =" field, a help icon, and "OK" and "Cancel" buttons.

What value would be returned by this function?

- (b) Fill in the information that would be needed to have *Excel* compute the probability that at most two of the patients survive.



The image shows the Excel BINOMDIST dialog box, identical to the one above. It has four input fields: "Number_s", "Trials", "Probability_s", and "Cumulative". Each field has a small icon to its right. To the right of each field is a label: "= number" for the first three and "= logical" for the last. Below the fields is a description: "Returns the individual term binomial distribution probability." and "Number_s is the number of successes in trials." At the bottom, there is a "Formula result =" field, a help icon, and "OK" and "Cancel" buttons.

What value would be returned by this function?

- (c) Find and interpret the value that would be returned by the *Excel* formula =BINOMDIST(4,4,0.9,FALSE).
(d) Find and interpret the value that would be returned by the *Excel* formula =BINOMDIST(3,4,0.9,TRUE).

23. The *p.d.f.* and *c.d.f.* of X , the relative humidity at a particular location are given below.

$$f_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 60x^3(1-x)^2 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^4(15 - 24x + 10x^2) & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Compute the probability that the relative humidity is at least 20%. Round your answer to four decimal places.

- (A) 0.0170
- (B) 0.3072
- (C) 0.6928
- (D) 0.9830
- (E) None of the above

24. Suppose that T is a uniform random variable on the interval $[0, 5]$.

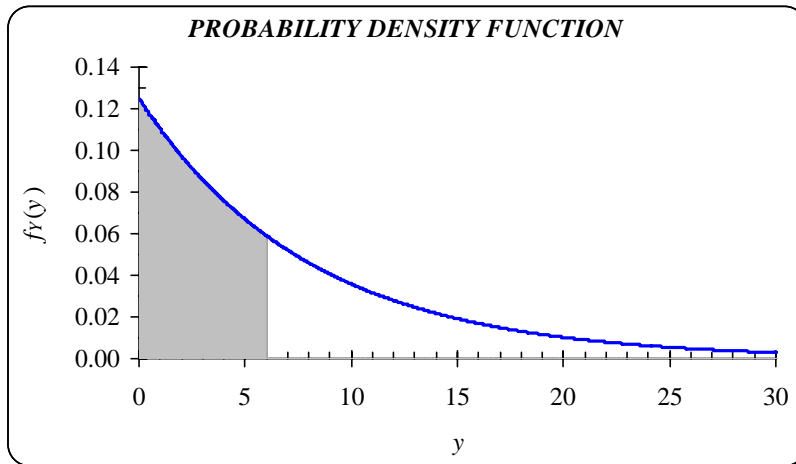
- (a) Find a formula for the *p.d.f.* of T .
- (b) Find $f_T(3)$.
- (c) Find $F_T(3)$.
- (d) Find $P(T = 3)$.

25. Suppose that you are waiting for your roommate to arrive home from an afternoon at the mall. She is equally likely to arrive home at any time between 4 pm and 6 pm. Let X be the continuous random variable that gives the time (in hours after noon) that she arrives home.

- (a) Find a formula for the *c.d.f.* of X .
- (b) Find the probability that your roommate arrives home between 4:45 pm and 5:15 pm.
- (c) Find the probability that your roommate arrives home before 4:15 pm.
- (d) Find the probability that your roommate arrives home at exactly 5:45 pm.

26. Suppose that a random variable, X , has a uniform distribution on the interval $[0, 20]$.
Compute μ_X .
- (A) 0
 - (B) 0.05
 - (C) 0.95
 - (D) 10
 - (E) 20
27. Let X be an exponential random variable with *p.d.f.* given by $f_X(x) = 0.2 \cdot e^{-0.2 \cdot x}$, for $x \geq 0$.
- (a) Find a formula for the *c.d.f.* of X .
 - (b) Use the result from Part (a) to compute $P(3.7 \leq X \leq 6.7)$. Round your answer to 4 decimal places.
 - (c) Find the value x_0 such that $P(X \leq x_0) = 0.5$.
 - (d) Find μ_X .
28. Let X be the continuous random variable that gives the time between the arrivals of consecutive customers at the checkout counter in a grocery store. It can be shown that X has an exponential distribution with a mean of 45 seconds.
- (a) Find the probability that the time between the arrivals of consecutive shoppers is between 1 and 2 minutes.
 - (b) Find the probability that the time between the arrivals of consecutive shoppers is at least 3 minutes.
 - (c) Find the probability that the time between the arrivals of consecutive shoppers is at most 45 seconds.
 - (d) Find and interpret $f_X(30)$.
 - (e) Find and interpret $F_X(30)$.

29. Let Y be an exponential random variable with parameter $\alpha = 8$. A graph of the $p.d.f.$ of Y is given below.



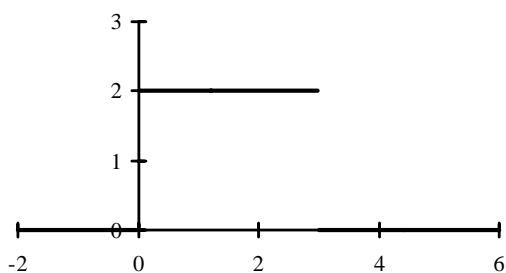
Which of the following expressions gives the area of the shaded region of the graph?

- (i) $f_Y(6)$
 - (ii) $F_Y(6)$
 - (iii) $P(Y < 6)$
- (A) (i) only
(B) (ii) only
(C) (iii) only
(D) (i) and (iii) only
(E) (ii) and (iii) only

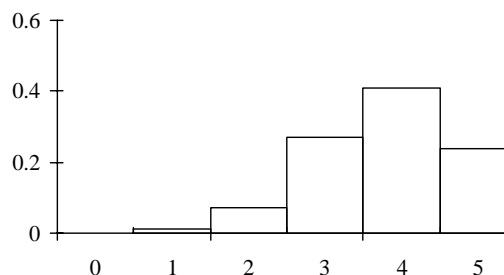
30. For each graph, select the statement that is most likely to be correct.

- i. The function is a *p.m.f.* for a finite random variable.
- ii. The function is a *p.d.f.* for a continuous random variable.
- iii. The function is a *c.d.f.* for a finite random variable.
- iv. The function is a *c.d.f.* for a continuous random variable.
- v. None of the above.

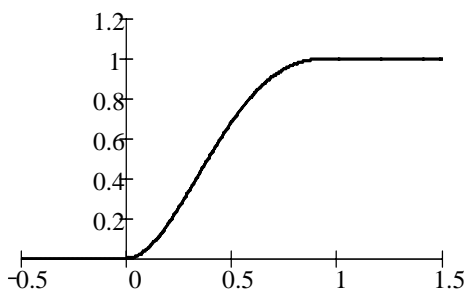
Hint: Look carefully at the numbers on the axes.



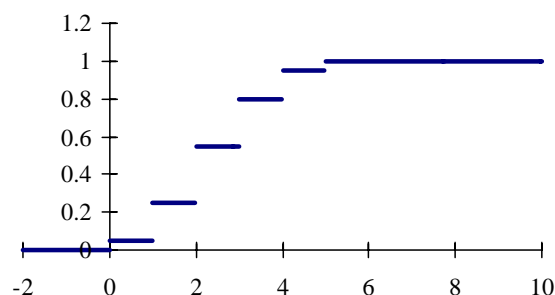
(a) _____



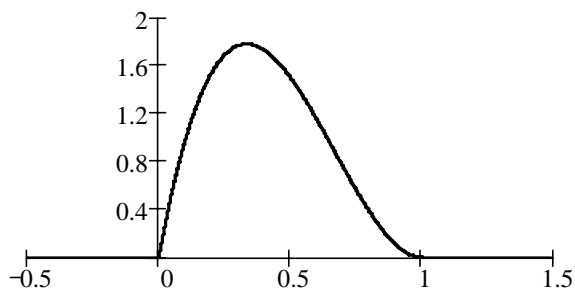
(b) _____



(c) _____



(d) _____



(e) _____

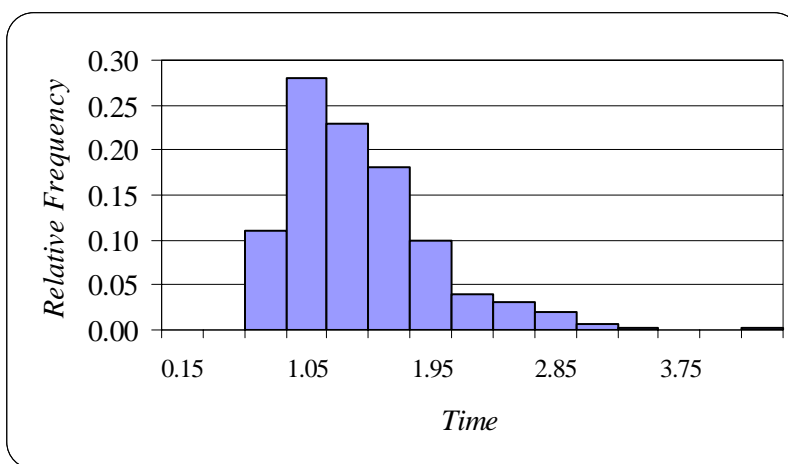
31. Eight students were asked how many hours per week they spend using a computer. The results are given below.

14, 20, 5, 10, 20, 9, 10, 21

- (a) Use this information to estimate the probability that a randomly selected student spends less than 15 hours per week using the computer.
 (b) Use this information to estimate the average number of hours per week that a randomly selected student spends using a computer.
32. Suppose that you conducted an experiment 100 times and obtained the following results for some continuous random variable, X .

Range	Frequency
$0 < x \leq 2$	9
$2 < x \leq 4$	15
$4 < x \leq 6$	36
$6 < x \leq 8$	25
$8 < x \leq 10$	15

- (a) Use this information to estimate $P(X > 6)$.
 (b) Draw a well-labeled histogram that approximates the graph of the *p.d.f.* of X .
33. Refer to the following plot of 500 weekly service times (in minutes) for an ATM.



- (a) Approximately how many service times were between 1.5 and 2.4 minutes?
 (b) Assuming that these service times are a random sample for the random variable S , which gives the length of time (in minutes) of an ATM transaction, estimate $P(S \leq 1.5)$.

34. Which of the following *Excel* formulas could be used to simulate one toss of a coin that is loaded in such a way that you are four times as likely to get a tail as you are to get a head?

- (A) =IF(RANDBETWEEN(1,2)<=2,"H","T")
- (B) =IF(RANDBETWEEN(1,2)=2,"H","T")
- (C) =IF(RAND()<=0.2,"H","T")
- (D) =IF(RAND()=0.2,"H","T")
- (E) None of the above

35. For each of the following functions, tell where it is located in *Excel*, and describe its use.

- (a) HISTOGRAM
- (b) IF
- (c) RAND
- (d) RANDBETWEEN
- (e) SORT
- (f) VLOOKUP