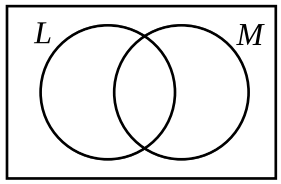
Business Mathematics I

TEST 1 STUDY GUIDE

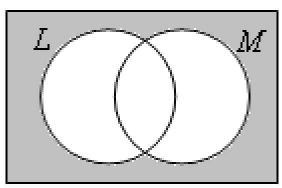
1. Consider a randomly selected new small business in your area. Let *L* be the event that it stays in business for the next 5 years and let *M* be the event that it is located in a shopping mall. Chamber of Commerce records yield the following estimates: $P(L) = 0.20$, $P(M) = 0.40$, and the probability of either *L* or *M* is 0.50.



(a) Shade the regions that represent the following events: (*i*) *L* and *M*. (*ii*) *M* but not *L*. (*iii*) $L \cap M^C$.

(b) Compute the following: (*i*) $P(L \text{ and } M)$. (*ii*) $P(M \text{ but not } L)$. (*iii*) The probability that neither *L* nor *M* happens. (*iv*) $P(M^C \cup L)$. (*v*) $P(L^C \cup M^C)$. (*vi*) $P(M \mid L)$. (*vii*) $P(L \mid M)$.

(c) Describe the shaded region (*i*) in words, and (*ii*) in set symbols.



(d) Are *L* and *M* independent? Are *L* and *M* mutually exclusive?

(e) What is the real-world interpretation of the statement "$P(L) = 0.2$"?

(f) Explain in real-world terms what that difference between $P(L)$ and $P(L \mid M)$ tells you about small businesses.

2. According to the department of tourism in a particular city, 35% of the tourists visit a museum, 59% of the tourists attend a sporting event, and 16% of the tourists both visit a museum and attend a sporting event

   (a) What percent of the tourists visit a museum or attend a sporting event?
   (b) What percent of the tourists do not visit a museum?
   (c) What percent of the tourists visit a museum but do not attend a sporting event?
   (d) What percent of the tourists either visit a museum or attend a sporting event, but do not do both?
   (e) What percent of the tourists do not visit a museum and do not attend a sporting event?
   (f) What percent of the tourists who attend a sporting event also visit a museum?

3. In a medical study of the common cold, 100 people with colds suffered the following symptoms:

   18 people had fevers,
   32 people had coughs,
   50 people had stuffy noses,
   5 people had both fevers and stuffy noses,
   4 people had both coughs and stuffy noses,
   7 people had both fevers and coughs, and
   2 people had all three symptoms.

   (a) If a person selected at random from the group had a stuffy nose, what is the probability that he or she also had a cough?
   (b) If a person selected at random from the group had a stuffy nose and a cough, what is the probability that he or she did not have a fever?

4. A flower shop has a bin of flower bulbs for sale, all of which look the same. A list of the bulbs that are in the bin is given below.

	Feathered Tulips	**Classic Tulips**
Lavender	15	13
Red	12	10
Yellow	7	19

   Suppose that a bulb is chosen at random.

   (a) What is the probability that the bulb is a Classic Tulip?
   (b) What is the probability that the bulb is a red Feathered Tulip?

5. On a blind date, there is a 35% chance that you'll like your date, a 95% chance that you'll like your date or the food, and a 15% chance that you'll like your date and the food. Find the probability that you'll like the food but not your date.

   (A) 0.05
   (B) 0.2
   (C) 0.25
   (D) 0.60
   (E) 0.75

6. You are an employer interested in hiring a new consultant. You would like to hire someone who has a business degree and three years of experience. When you contact a hiring agency, you are given the following information about their pool of potential employees:

   65% have a business degree.
   30% have three years of experience.
   25% have neither.

   What fraction of this pool of potential employees fulfills your criteria?

   (A) 10%
   (B) 19.5%
   (C) 20%
   (D) 25%
   (E) 45%

7. Which of the following statements must always be true about events *A* and *B*?

   $(i)\ P(A \mid B) + P(A \mid B^C) = 1$
   $(ii)\ P(A \cap B) = P(A) + P(B) - P(A \cup B)$
   $(iii)\ P(A \cap B) = P(A) \cdot P(B)$

   (A) (*i*) only
   (B) (*ii*) only
   (C) (*iii*) only
   (D) (*i*) and (*ii*) only
   (E) None of the above

8. A fair die is rolled twice and the number of spots on the upper face is recorded for each roll. We can take all possible pairs of numbers, each of which is between 1 and 6, as our sample space, *S*. Since the die is fair, we will assume that each of the 36 outcomes is equally likely, and therefore, that each has the same probability.

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Let *F* be the event that the sum of the faces is less than 5, and let *G* be the event that the product of the faces is even. Which of the following statements is/are true?

(*i*) *F* and *G* are not mutually exclusive because $P(F \cup G) \neq P(F) + P(G)$.
(*ii*) *F* and *G* are not mutually exclusive because $F \cap G \neq \varnothing$.
(*iii*) *F* and *G* are mutually exclusive.

(A) (*i*) only
(B) (*ii*) only
(C) (*iii*) only
(D) (*i*) and (*ii*) only
(E) None of the above

9. Evaluate $\sum_{k=1}^{4}(k^2+4)$.

10. Evaluate $\sum_{k=1}^{100}(2k+1) - \sum_{k=1}^{99}(2k+1)$.

(A) 0
(B) 1
(C) 200
(D) 201
(E) None of the above

11. Use summation notation to rewrite $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{12}{13}$.

12. Re-index $\sum_{k=5}^{9}(3k+7)$ so that the index of summation is *i*, where *i* starts at 1.

13. Re-index $\sum_{k=4}^{7}(k^2+3k+2)$ so that the index is *i*, where *i* ranges from 2 to 5.

(A) $\sum_{i=2}^{5}(i^2-i-8)$

(B) $\sum_{i=2}^{5}(i^2+7i+8)$

(C) $\sum_{i=2}^{5}(i^2+7i+12)$

(D) $\sum_{i=2}^{5}(i^2+3i+12)$

(E) None of the above.

14. Given $\sum_{i=1}^{5}a_i=121$ and $\sum_{j=1}^{5}b_j=12$, find $\sum_{k=1}^{5}(3b_k-a_k)$.

15. A spinner stops at numbers 1, 2, or 3 with the probabilities 0.5, 0.3, and 0.2; respectively. You are to make two independent spins and record the numbers. Let *X* be the random variable which gives the sum of the numbers obtained on the two spins.

(a) Set up a sample space, *S*, for this experiment, and assign realistic probabilities to each of the nine outcomes.
(b) Compute $P(2<X\le 5)$.
(c) Compute $P(X=6)$.
(d) Compute $E(X)$.
(e) Compute $\sum_{x=2}^{4}x^2\cdot P(X=x)$.
(f) What is the real-world interpretation of $E(X)$?

16. Suppose that you have three nickels and five quarters in your left pocket and that you have two nickels and six quarters in your right pocket. You randomly select one coin from each pocket.

    (a) Set up a sample space, *S*, for this experiment, and assign realistic probabilities to each of the four outcomes.
    (b) Find the probability that you select at least one quarter.
    (c) Find the probability that you select a total of 30 cents from your pockets.
    (d) Find the probability that the total amount of money selected from your pockets is less than 25 cents.
    (e) Find the expected value of the total amount of money selected from your pockets.

17. Let *X* be the amount of money that an investment returns. Which of the following is a correct interpretation of $E(X)$?

    (A) A possible return on the investment
    (B) The average return on the investment
    (C) The maximum return on the investment
    (D) The most likely return on the investment
    (E) None of the above

18. Let *X* be a random variable that takes on the values 1 and 10. If $E(X) = 2.1$, then which of the following must be true?

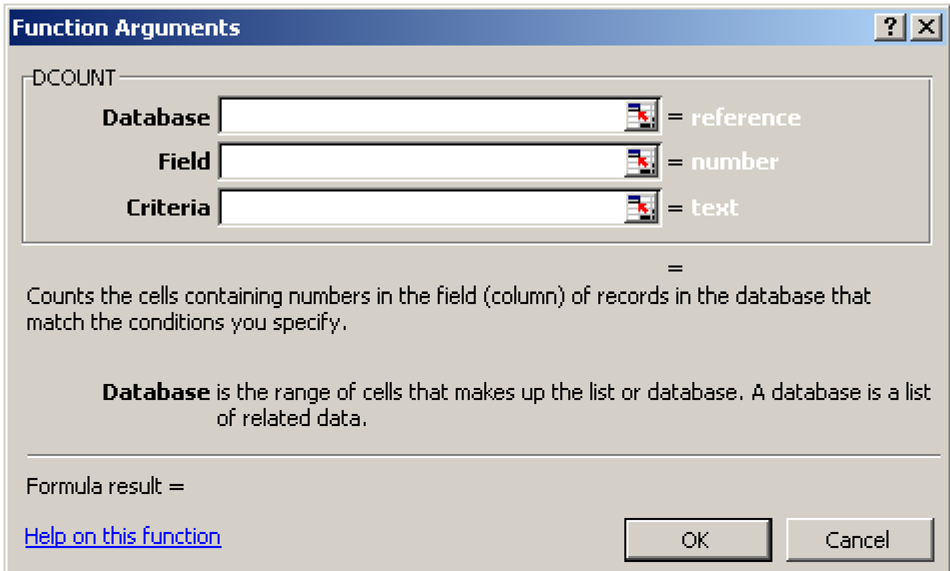
    (A) $P(X = 10) < P(X = 1)$
    (B) $P(X = 10) > P(X = 1)$
    (C) $P(X = 10) = P(X = 1)$
    (D) $P(X = 10) = 0$
    (E) None of the above

19. An excerpt from the incoming flights at Chicago's O'Hare International Airport is given below.

	A	B	C	D	E	F
1	**Airline**	**Flight**	**Schedule**	**Gate**	**City**	**Remarks**
2	American Airlines	1081	9:47	K9	BOSTON	ON TIME
3	American Airlines	1484	8:15	H6	SAN DIEGO	9:31
4	American Airlines	581	9:04	K17	TAMPA/ST. PETE	9:29
5	America West	721	9:52	E8	LAS VEGAS OAKLAND	ARRIVED
6	Continental Airlines	3300	11:41	E12	HOUSTON	11:55
7	Continental Airlines	2408	7:54	E14	CLEVELAND	ON TIME
8	Continental Airlines	1179	7:56	E4	NEW YORK/NEWARK	ON TIME
9	Independence Air	1908	8:47	E10	WASHINGTON DULLES	ON TIME
10	Continental Airlines	2337	9:30	E4	CLEVELAND	ON TIME
11	Northwest Airlines	144	11:14	E7	MINNEAPOLIS	ON TIME
12	United Airlines	9985	9:52	B4	ANDREWS AFB	ARRIVED
13	Air Canada	4625	7:00	F9	CINCINNATI	ARRIVED
14	Lufthansa	5629	7:00	F9	CINCINNATI	ARRIVED
15	Air Canada	5782	9:51	C19	DENVER	ARRIVED
16	United Airlines	7765	8:05	C4	HARTFORD SPRINGFIE	ARRIVED
17	United Airlines	122	10:13	B11	LOS ANGELES	11:13
18	Air Canada	5052	10:13	B11	LOS ANGELES	11:13
19	Air Canada	4748	8:35	F12	MONTREAL	10:30
20	United Airlines	1526	11:01	B21	PHOENIX	11:55
21	United Airlines	2166	9:32		PITTSBURGH	ON TIME
22	United Airlines	7144	8:14	F7	ROCHESTER	ARRIVED
23	US Airways	1263	11:56	F8	PHILADELPHIA	11:30
24	US Airways	1825	11:09	F8	WASHINGTON DC	ON TIME

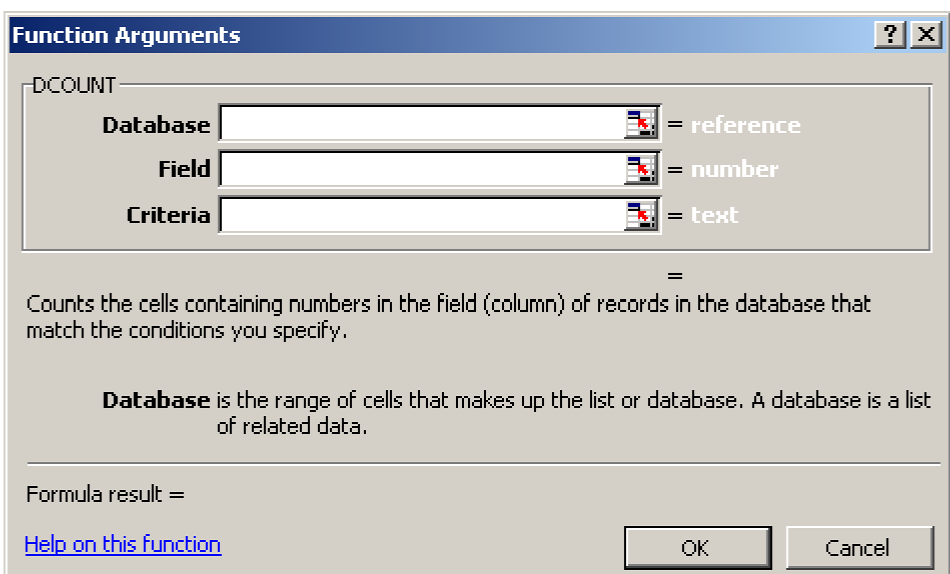
(a) Enter the information that would be needed in cells H2:M2 of the worksheet as well as the dialog box to have *Excel* count the number of United Airlines flights.

	H	I	J	K	L	M
1	**Airline**	**Flight**	**Schedule**	**Gate**	**City**	**Remarks**
2						



(b) Enter the information that would be needed in cells H5:M6 of the worksheet as well as the dialog box to have *Excel* count the number of United Airlines flights that were originally due to arrive before 10:00.

	H	I	J	K	L	M
5	**Airline**	**Flight**	**Schedule**	**Gate**	**City**	**Remarks**
6						



(c) Find and interpret the value that would be returned by the *Excel* formula =DCOUNT($A$1:$F$24,,$H$9:$M$10).

	H	I	J	K	L	M
9	**Airline**	**Flight**	**Schedule**	**Gate**	**City**	**Remarks**
10						ON TIME

(d) Find and interpret the value that would be returned by the *Excel* formula =DCOUNT($A$1:$F$24,,$H$14:$M$15).

	H	I	J	K	L	M
14	**Airline**	**Flight**	**Schedule**	**Gate**	**City**	**Remarks**
15	Continental Airlines					ON TIME

(e) Use the records to estimate the probability that a Continental Airlines flight is on time.

20. Let F be the event that a person is unemployed and let H be the event that a person has a high school diploma. What does the conditional probability $P(H \mid F)$ represent?

(A) The probability that a randomly chosen person who is unemployed has a high school diploma.
(B) The probability that a randomly chosen person with a high school diploma is unemployed.
(C) The probability that a randomly chosen person is unemployed and has a high school diploma.
(D) The probability that a randomly chosen person is unemployed.
(E) None of the above

21. Let J be the event that a traffic accident is caused by poor judgment and let F be the event that the driver at fault is female. Records show that $P(J) = 0.30$ and $P(F) = 0.50$. If J and F are independent, what is $P(J^C \cap F)$?

(A) 0
(B) 0.15
(C) 0.20
(D) 0.35
(E) 0.50

22. A box contains a dozen items of which three are defective. Two items are drawn from the box, without replacement. Find the probability that both are non-defective.

23. A fair die is rolled twice and the number of spots on the upper face is recorded for each roll. We can take all possible pairs of numbers, each of which is between 1 and 6, as our sample space, *S*. Since the die is fair, we will assume that each of the 36 outcomes is equally likely, and therefore, that they all have the same probability.

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Let *F* be the event that the sum of the faces is greater than or equal to 8 and let *G* be the event that the sum of the faces is even. Which one of the following statements is true?

(A) *F* and *G* are independent because $P(G \mid F) = P(G)$.
(B) *F* and *G* are independent because $P(F \cap G) = P(F) \cdot P(G)$.
(C) *F* and *G* are independent because $P(F \cap G) = 0$.
(D) *F* and *G* are independent because $P(F \cap G) \neq 0$.
(E) None of the above

24. The following two-way table contains information about bikes that were recently taken to a bike shop for repair.

Model	Defective Chain	Non-defective Chain	Total
Dahon	20	37	57
Breezer	28	15	43
Total	48	52	100

Use the information in the table to answer the following.

(a) Represent the events and probabilities for this problem in a tree diagram.
(b) Estimate the probability that a randomly chosen bike has a defective chain.
(c) Use Bayes' Theorem to find the probability that a randomly chosen bike with a defective chain is a Breezer.
(d) Verify your answer to Part (c) directly from the table.

25. A printer has three bookbinding machines. The table below gives the proportion of the total book production for each machine and the probability that the machine produces a defective binding.

Machine	**Proportion of books bound**	**Probability of defective bindings**
1	0.4	0.03
2	0.1	0.06
3	0.5	0.01

(a) What percent of the books have defective bindings?
(b) What percent of the books with defective bindings are bound by Machine 2?

26. A researcher surveys patients who use a hospital emergency room. The researcher finds that 30% of the patients require hospitalization. Of those patients, 60% have medical insurance. Of the patients who do not require hospitalization, 65% have medical insurance. Let H be the event that a randomly chosen patient requires hospitalization and let M be the event that a randomly chosen patient has medical insurance.

(a) Compute $P(M)$.
(b) Compute $P(H \mid M)$.

27. As accounts manager in your company, you classify 80% of your customers as “good credit” and the others as “risky credit”. Records show that 50% of the accounts held by customers in the “risky credit” category are overdue, but only 10% of the accounts held by customers in the “good credit” category are overdue.

(a) What percent of the accounts are overdue?
(b) What percent of the accounts that are overdue are held by customers in the “risky credit” category?

28. If two events E and F form a partition of a sample space, S, then which of the following statements ***must*** be true?

    (i) E and F are independent.
    (ii) $E \cap F = \varnothing$.
    (iii) $E \cup F = S$.

    (A) (i) only
    (B) (ii) only
    (C) (iii) only
    (D) (i) and (iii) only
    (E) (ii) and (iii) only

29. Suppose that $P(E)$, $P(F)$, $P(A \mid E)$, and $P(A \mid F)$ are known. Which of the following statements ***must*** be true in order to use Bayes' Theorem to calculate $P(E \mid A)$ from this information?

    (i) The union of E and F is S.
    (ii) E and F are mutually exclusive.
    (iii) E and F are independent.

    (A) (ii) only
    (B) (iii) only
    (C) (i) and (ii) only
    (D) (i) and (iii) only
    (E) (i), (ii), and (iii)

30. The first 20 records in the file ***Loan Records.xlsx*** are given below.

Bank Information		**Borrower**			**Result**
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
1	Cajun		Bachelor's Degree		no
2	BR	11			no
3	BR	13			yes
4	Cajun		Graduate Degree		no
5	DuPont			Normal	no
6	BR	20			no
7	BR	1			yes
8	DuPont			Boom	no
9	BR	7			no
10	BR	12			yes
11	DuPont			Normal	no
12	Cajun		Bachelor's Degree		no
13	BR	7			no
14	DuPont			Normal	yes
15	BR	16			yes
16	BR	18			yes
17	Cajun		Graduate Degree		yes
18	DuPont			Boom	no
19	DuPont			Recession	yes
20	BR	8			yes

Let C be the event that the current state of the economy is normal.

Use the data to estimate the following probabilities.

(a) $P(S)$
(b) $P(S \mid C)$
(c) $P(C \mid S)$