Student Name: Key

[1 POINT].

Section #: 1+5

[1 POINT]

Math 122B Exam 1
[2.1 – 3.2]

Bridges

Oct 3, 2016

POINTS ARE AWARDED BASED ON YOUR USE OF **CALCULUS** AND **ALGEBRA**. YOUR FINAL ANSWER MUST FOLLOW FROM YOUR WORK. UNLESS YOU ARE TOLD TO DO OTHERWISE, A SOLUTION BASED ONLY ON A GRAPH OR BY USE OF YOUR CALCULATOR WILL RECEIVE NO CREDIT. YOU MUST CLEARLY AND LOGICALLY SHOW ALL WORK. ANSWERS ARE TO BE **EXACT** AND IN **SIMPLEST FORM** UNLESS YOU ARE TOLD TO DO OTHERWISE.

YOU MUST FOLLOW ALL INSTRUCTIONS!

B

1. The table below gives the position (p) of a particle as a function of time (t) in years, where p is in meters. Assume $p(t)$ is continuous and differentiable for $0 < t < 8$.

t	0	2	4	6	7
$p(t)$	1	−9	−1	5	11

10 PTS

(a) Calculate the best estimate for the instantaneous velocity of the particle at $t = 4$.

2 3) $p'(3) \approx \frac{-1+9}{2} = \frac{8}{2} = \boxed{4}$ ↓ vel

2 3) $p'(5) \approx \frac{5+1}{2} = \boxed{3}$

2 2) $p'(4) \approx \frac{4+3}{2} = \boxed{3.5}$ m/yr

(b) Based on the table, do you think the graph of $p(t)$ is concave up or concave down in the interval $(2, 6)$? In one good sentence, explain your reasoning.

4 2) Slopes of tangent lines are decreasing, ∴ $p(t)$ is c. down on $(2, 6)$

2. For some drugs, the size of the dose, D (in mL), is a function of the person's weight, w (in kg), so that $D = f(w)$. What is the practical interpretation of the statement $f'(75) = 2$? Circle the letter of the best answer listed below.

8 PTS

(a) When the weight is 75 kg, the dose is 2 mL.

(b) If the dose increases from 75 mL to 76 mL, then the person's weight increases by 2 kg.

(c) If a person's weight decreases from 75 kg to 74 kg, then the person's dose decreased by 2 ~~kg~~ mL.

(d) If the weight decreases from 75 kg to 74 kg, then the dose is 2 mL. D

3. Find the value of c where the line L, tangent to the graph of f at (0,2), intersects the x-axis.
Notice that line L contains the point (5, 4).

8 PTS

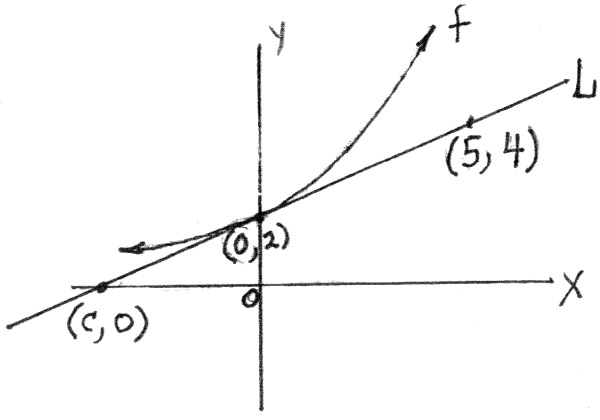
$m = \frac{4-2}{5} = \frac{2}{5}$ • +3

$y = \frac{2}{5}x + 2$

$0 = \frac{2}{5}c + 2$ • +3

$-2 = \frac{2}{5}c$ • +2

$\boxed{c = -5}$



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B

4. Use the limit definition of the derivative and algebra to find the derivative of the function below. You may not use any shortcut differentiation rules on this problem. Show all work and every step.

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$$g(x) = \frac{7}{3x-2}$$

$$g'(x) = \lim_{h\to 0}\left(\frac{\frac{7}{3(x+h)-2} - \frac{7}{3x-2}}{h}\right) \quad +4$$

$$= \lim_{h\to 0} \frac{21x-14-7(3x+3h-2)}{h(3x+3h-2)(3x-2)} \quad +6$$

$$= \lim_{h\to 0} \frac{21x-14-21x-21h+14}{h(3x+3h-2)(3x-2)} \quad +9$$

$$g'(x) = \frac{-21}{(3x-2)^2}$$

$9x^2-12x+4$

5. Find the equation of the line tangent to the graph of the function below at $p = 2$. Put your final answer in ~~slope-intercept form~~. POINT-SLOPE FORM.

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$f(p) = -7p^2 + 5p$

$f'(p) = -14p + 5 \quad (+2)$

$f'(2) = -28 + 5 = -23 = m \quad (+3)$

$f(2) = -7(4) + 5 \cdot 2$

$= -28 + 10$

$= -18 \quad (+3) \quad (2, -18)$

$Y + 18 = -23(X - 2) \quad (+3)$

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NOTE: PROBLEMS FROM THIS PAGE COME FROM SECTION 2.6

6. $W(x) = \begin{cases} 3x^{-2} + \frac{1}{9}x & \text{if } x \le 3 \\ -\frac{1}{9}x + 1 & \text{if } x > 3 \end{cases}$

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(a) ~~Consider the function, W, above with constant A.~~ Using calculus and algebra, determine if W is continuous at $x = 3$. SHOW ALL WORK AND STATE YOUR CONCLUSION.

$W_1(3) = 3 \cdot \frac{1}{9} + \frac{1}{9} \cdot 3 = \boxed{\frac{2}{3}}$ (+2) (3

$W_2(3) = -\frac{1}{9} \cdot 3 + 1 = \boxed{\frac{2}{3}}$ (+1) OR (2

YES (2

(b) Using calculus and algebra, determine if W is differentiable at $x = 3$.
SHOW ALL WORK AND STATE YOUR CONCLUSION.

$W_1'(3) = -6x^{-3} + \frac{1}{9} = -\frac{6}{27} + \frac{1}{9} = \boxed{-\frac{1}{9}}$ (2

$W_2'(3) = \boxed{-\frac{1}{9}}$ ← (+1) OR (2

YES (C) Is W(x) A DIFF'ABLE FUNCTION? (+1) (1

7. Consider the function $g(x) = x^3 - 2x^2 - 4x + 5$. Use your knowledge of calculus to determine the solutions to the questions below. You must show all work – a solution based only on a graph or on your knowledge of polynomial functions will not be accepted.

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(a) On what interval(s) is g decreasing?

$g'(x) = 3x^2 - 4x - 4$ +2

$(3x + 2)(x - 2) = 0$ +3

$x = -\frac{2}{3}, \; x = 2$

+6

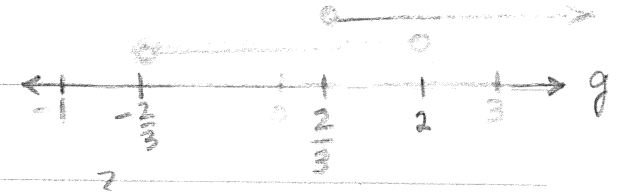
$\boxed{\left(-\frac{2}{3}, 2\right)}$

(b) On what interval(s) is g concave up?

$g''(x) = 6x - 4 > 0$

$x = \frac{4}{6} > \frac{2}{3}$

$\boxed{\left(\frac{2}{3}, \infty\right)}$



+4

?

(c) On what interval(s) is g both decreasing and concave up?

$\boxed{\left(\frac{2}{3}, 2\right)}$

+2

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B

8. Use the Rules of Calculus that we have studied to find the first derivative of each of these functions.

6→ (a) $g(x) = x^{-3/2} - \sqrt{7}\,x^{5/2}$

CHAP 3→ $g'(x) = -\frac{3}{2}x^{-5/2} - \frac{5}{2}\sqrt{7}\,y^{3/2}$

6→ (b) $k(y) = \frac{e}{y} + 4 \cdot 5^{y}$

$K(y) = ey^{-1} + 4 \cdot 5^{y}$

CHAP 3→ $K'(y) = -ey^{-2} + 4(\ln 5)5^{y}$

6→ (c) $P(m) = m^5(3 - \sqrt{m})$

$P(m) = 3m^5 - m^{11/2}$

CHAP 2→ $P'(m) = 15m^4 - \frac{11}{2}m^{9/2}$

YOU CAN DO THIS ONE.

6→ (d) $\beta(c) = \frac{c^5 - 3c}{5c^4} + 14^c$

$\beta(c) = \frac{1}{5}c - \frac{3}{5}c^{-3} + 14^c$

CHAP 3→ $\beta'(c) = \frac{1}{5} + \frac{9}{5}c^{-4} + \ln(14) \cdot 14^c$

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