#19 PRACTICE TEST 2 [2.3-3.6]

POINTS ARE AWARDED BASED ON YOUR USE OF CALCULUS. SHOW YOUR WORK LOGICALLY AND CLEARLY, GIVE EXACT AND SIMPLEST ANSWERS UNLESS STATED OTHERWISE. PLEASE CIRCLE OR BOX IN YOUR FINAL ANSWER.

YOU MAY NOT USE A CALCULATOR ON PROBLEMS 1 – 3 (THE FIRST 2 PAGES OF THIS TEST).

1. Find the first derivative of each of these functions. [8 POINTS EACH]

(a) $f(a) = \cos(a^2) \cdot a^{-2}$

(b) $g(t) = [\arctan(5t^2)]^{-1}$

(c) $W(z) = \left(z^3 - \sqrt[3]{z}\right) \cdot \ln(z)$

(d) $H(y) = 3^{tan\left(\frac{1}{y}\right)}$

2. A magnetic field, M is given as a function of the distance, d from the center of a wire according to the piecewise function below where A and C are positive constants:

$$M(d) = \begin{cases} A \cdot \frac{d}{C} & for\ d < C \\ A \cdot \frac{C}{d} & for\ d \geq C \end{cases}$$

Use the definition of the derivative to show that M is not differentiable at $d = C$. Show all steps. [5 POINTS]

3. On what interval is $y = \ln(1 + x^2)$ concave down? Show all steps. [9 POINTS]

4. Use the chart below to answer problems (a) – (d). [7 POINTS EACH]

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
-2	3	-7	5	7	1	0
5	-1	5	0	-6	2	4
4	9	-2	3	-2	1	5
-6	8	6	-5	-3	6	7

(a) Let $h(x) = f(x)/g(x)$.

(i) Find the value of $h'(5)$. SHOW WORK.

(ii) Is $h(x)$ increasing or decreasing at $x = 5$ and why?

(b) Let $h(x) = (f(x))^2$

(i) Find the value of $h''(-6)$. SHOW WORK.

(ii) Is $h(x)$ concave up or concave down at $x = -6$ and why?

(c) Let $h(x) = f[g(x-1)]$. Find the value of $h(5)$. SHOW WORK.

(d) Let $h(x) = g(x) \cdot f(x)$. Find the equation of the line tangent to h at $x = -2$.
SHOW ALL WORK.

5. Use the identity $10^{\log(x)} = x$ to prove that $\frac{d}{dx}(\log(x)) = \frac{1}{(ln10)x}$. Show all steps. [7 POINTS]

6. The temperature, T, in degrees Fahrenheit, of a cold yam placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven.
CIRCLE THE CORRECT ANSWER TO THE QUESTIONS BELOW. [2 POINTS EACH ANSWER]

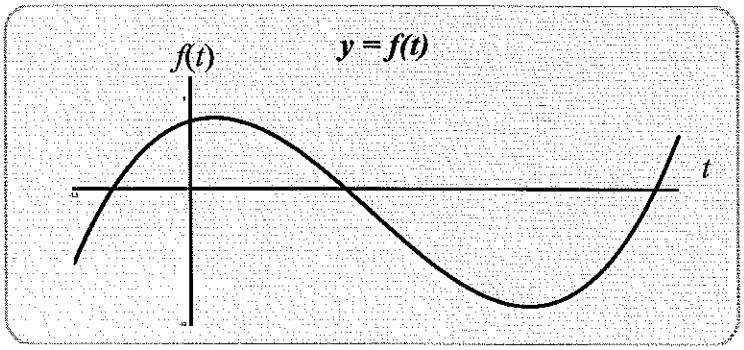
(a) What is the sign of $f'(t)$?
(i) positive
(ii) negative

Why?
(i) The temperature of the oven is decreasing over time.
(ii) The time is increasing.
(iii) The temperature of the yam is increasing over time
(iv) The temperature of the oven is increasing over time.

(b) What are the units of $f'(20)$?
- (i) min/°F
- (ii) none of these
- (iii) min
- (iv) °F
- (v) °F/min

(c) What is the practical meaning of the statement $f'(20) = 1.9$?
- (i) At time $t = 20$ minutes, the temperature T has increased by 1.9 °F
- (ii) At time $t = 1.9$ minutes, the temperature T has increased by 20 °F
- (iii) At time $t = 1.9$ minutes, the temperature T increases by approximately 20 °F for each additional minute in the oven.
- (iv) At time $t = 1.9$ minutes, the temperature T is 20 °F.
- (v) At time $t = 20$ minutes, the temperature T increases by approximately 1.9 °F for each additional minute in the oven.

7. The function shown below gives the position of a particle at time .
ANSWER EACH QUESTION IN THE BLANK GIVEN – IF THERE IS MORE THAN ONE ANSWER, THEN YOU MUST LIST THEM ALL. [2 POINTS EACH BLANK]



THE HORIZONTAL AXIS OF THIS GRAPH BEGINS AT -3 AND ENDS AT ALMOST 13. THE POINTS LABELED WERE: -1, 1.5, 7, 10, 11.

(a) At what labeled point(s) is the position of the particle positive?

_____

(b) At what labeled point(s) is the velocity of the particle negative?

_____

(c) At what labeled point(s) is the acceleration of the particle positive?

_____

(d) At what labeled point(s) is the position of the particle increasing?

_____

(e) At what labeled point(s) is the velocity of the particle decreasing?

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