More Applications of Exponential and Logarithmic Functions and Equations

I. The STARBUCKS Problem

The first Starbucks opened in 1971 at Pike Place Market in Seattle. The chart below shows the growth of the company from the year 1988.

Year	1988	1990	1992	1994	1996	1998	2000	2002
Number	33	84	165	425	1015	1886	3501	5886

1. On your calculator, create a scatterplot of the data, with the year representing number of years from 1988; i.e. enter 1988 as 0, 1990 as 2, etc.

2. What type of function would seem to best model the growth of Starbucks? ____________________________

3. Using the model $n(t) = a \cdot b^t$, algebraically find a function that fits the data. (You need to find values for "a" and "b". Round the value of b to 3 decimal places.) Show your work. On your calculator, graph n(t) to see how it fits the scatterplot.

n(t) = ____

4. Use **ExpReg** (Exponential Regression) to find the exponential function n(t) that "best fits" the data. On your calculator, graph n(t).

n(t)= ____

5. Using the exponential regression function from part 4 above, find the projected number of Starbucks in the year 2004?

n(______) = ________________________________

6. Using the exponential regression function, find when the number of Starbucks will reach 11,000. Show your work algebraically and check your answer graphically.

t = ____________________

II. The Exponential Growth/Decay Model: Many natural phenomena have been found to follow the law that an amount A varies with time t according to:

$$\boxed{A(t) = A_0 e^{kt}}$$

Here, A_0 is the initial amount (t=0), and $k \neq 0$ is a constant. If $k > 0$, then the equation states that the amount A is increasing over time (exponential growth); if $k < 0$, the amount A is decreasing over time (exponential deay).

1. **Bacterial Growth:** A colony of bacteria grows according to the exponential growth model. Initially, there are 100 grams of the bacteria, and 5 days later there are 125.2 grams.

a. Find the function A(t) that models the growth of the bacteria.
(Hint: The given information tells you the value of A_0, and you have enough information to find k.)

A(t) = ______________________

b. What does the value of k represent to the problem? ____

c. Use your model to determine how long it will tke for the population to reach 140 grams.

t = ___________

2. **Radioactive Decay**: The half-life of radium is 1690 years. (Note: The half-life is the time it takes for half of the substance to decay.) If 10 grams are present now, how much will be present in 100 years? In 1000 years? (Hint: You need to find k first!)

III. The Turkey Problem: Newton's Law of Cooling equation states that the temperature of a heated object decreases over time toward the temperature of the surrounding environment. The temperature A(t) of a heated object at a given time t can be modeled by the function:

$$\boxed{A(t) = C + (A_0 - C)e^{r \cdot t},}$$

where A_0 represents the initial temperature, and C represents the temperature of the "environment".

Suppose that a roast turkey is taken from an oven when its temperature has reached 185 degrees F and is placed on a table in a room where the temperature is 75 degrees. Also, suppose that the temperature of the turkey is 150 degrees after 30 minutes.

1. Use this information to find the function A(t). (Round the value of r to 5 decimal places.)

A(t) = ____

2. Graph A(t) on your calculator. Show your graph below. Label the y-intercept and the horizontal asymptote.

3. What is the temperature after 45 minutes? ________________

4. When will the turkey have cooled to 100 degrees? Solve algebraically, and check your answer graphically.

t = ____________

IV. Cooling Time of Pizza (Newton's Law of Cooling)

A pizza baked at $450^{\circ}F$ is removed from the oven at 5:00 PM and placed in a room where the temperature is $70^{\circ}F$. After 5 minutes, the pizza is at a temperature of $300^{\circ}F$.

1. Find the function A(t) that would tell the temperature of the pizza for any time t (in minutes) after 5:00 PM. Show your work.

A(t) = ____

2. Use your function A(t) and algebraically determine what time you can begin eating the pizza if you want its temperature to be $135^{\circ}F$. Show your work.

Time is ________________ PM

V. Earthquake Problem: You can gauge the amount of energy released by an earthquake by its Richter magnitude, devised by Charles Richter in 1935. The Richter magnitude is a base-10 logarithmic function of the energy released by the earthquake. The following data show Richter magnitude m for earthquakes that release energy equivalent to the explosion of x tons of TNT.

x (tons)	m (Richter magnitude)
1,000	4.0
1,000,000	6.0

1. Algebraically find the particular equation of the common logarithmic function $m = a + b \cdot \log x$ that fits the two points. Show your work.

m = ________________________________

2. Use the equation to predict the the Richter magnitude for:

a. The 1964 Alaska earthquake, one of the strongest on record, that released the energy of 5 billion tons of TNT.

A Richter magnitude of ___________________

b. The Chilean earthquake of 1960 had a Richter magnitude of about 9.0. How many tons of TNT would it take to produce a shock of this magnitude? Solve this problem algebraically. Write your answer in scientific notation.

__________________________ tons

VI. Ball Bounce Problem: A ball is dropped from a height of 2.65 feet and the maximum height of the first eight bounces are measured by a motion detector and recorded in the table below. (Bounce #0 represents the initial drop height.)

Bounce Number	0	1	2	3	4	5	6	7	8
Bounce Height	2.65	2.09	1.66	1.35	1.11	0.92	0.73	0.59	0.50

1. Enter the data into the lists of your calculator and create a scatterplot of this data. Sketch it below. Label your axes to show approximate x and y values.

3. The model for this bouncing ball data is an ***exponential function.*** We will attempt to fit this data with an exponential function of the form:

$$y = a \cdot b^x$$

where **x** represents the bounce number and **y** represents the corresponding rebound height. Explain what the value of **a** represents to the problem and record this value below.

a represents ____

a= ____________

4. To find the value of **b**, select another ordered pair from your scatterplot, substitute the values for **x** and **y** into the exponential equation (along with the value of **a**), and solve for **b**. (You should select a point not close to the first point.) Show your work below and record your final function. (Round the value of b to the nearest hundredth. Then graph the function to see how it fits the data.

y = __________________________________

5. The value of **b** also has a "real world" meaning to the problem. Explain what the value of **b** represents to the problem situation. Be specific!

b represents ____

6. As a check, perform an *exponential regression* (**ExpReg**) on your data which will allow your calculator to find the best-fitting exponential function through the set of data. The values you found for **a** and **b** in parts 3 and 4 should be similar to the values from the exponential regression. Again, round all values to the nearest hundredth and record this equation below.

y = ____________________________________

7. Rewrite the regression equation using the words "Bounce Number" and "Rebound Height", instead of the variables **x** and **y**.

____

8. Using the exponential regression model in part 6 above, write an equation to determine the smallest number of bounces required for the rebound height to be less than 10% of its starting/drop height. Solve this equation algebraically. Show all of your work. When writing your final answer, remember that the number of bounces must be an integer value.

Bounce number ___________