Examples:

1. One leg of a right triangle is 6 cm longer than the other leg. Express the hyotenuse of the triangle in terms of the short leg.

2. Express the circumference C of a circle as a function of the area A of the circle.

3. The surface area of a cylinder is 100 square centimeters. Express the height H of the cylinder as a function of the radius R.

Note: Do your work on the next example on another sheet of paper.

4. A farmer with 5,000 feet of fencing wants to enclose a rectangular field and then divide it into two plots by adding a fence in the middle parallel to one of the sides.

   a. Sketch a picture of the problem.

   b. What is the area of the field if the width (one of the sides parallel to the added fence) is 500 ft?

   c. What is the area of the field if the width is 1200 ft?

   d. If the width of the pasture is x ft, find an algebraic representation A(x) for the area of the field.

   e. Using the function A(x), graphically find the dimensions (length and width) of the field that would maximize its area.

   f. If the farmer wanted the area of the field to be 700,000 square feet, what would be the dimensions of the field?

Problems:

5. A rectangle has an area of 56 square feet. Determine the perimeter P as a function of the length L.

6. The volume of a cylindrical can is 100 cm^3. Express the radius R of the can as a function of the height H.

7. A ladder of length 8 feet leans against a wall. Express the height of the top of the ladder as a function of the distance x between the foot of the ladder and the base of the wall.

Note: Answer the following problems on another sheet of paper.

8. A rectangle has a perimeter of 200 in.

  a. If the width is 10 in, what would the length of the rectangle be? What would the area of the rectangle be?

  b. If the width is 30 in, what would the length of the rectangle be? What would the area of the rectangle be?

  c. If the width of the rectangle is x in, write an algebraic representation A(x) for the area of the rectangle.

  d. Using the function A(x), graphically find the dimensions of the rectangle that would maximize the area of the rectangle. Express in words, what your answer to this problem tells you.

9. Twenty-five feet of fence is to be put around a garden. The plans have one edge of the garden to be along the side of a house, with the fence enclosing the other three sides of a rectangle.

  a. Draw and label a figure that illustrates the problem. Assume that the width of the garden is the side perpendicular to the house, and the length of the garden is the side parallel to the house.

  b. If the width of the garden is 4 feet, what is the length of the garden? What is the area of the garden?

  c. If the width of the garden is 9 feet, what is the length of the garden? What is the area of the garden?

  d. If the width of the garden is x feet, write an algebraic representation for the length of the garden in terms of the width x. Write a function A(x) which represents the area of the garden as a function of the width x.

  e. Graph y = A(x) and find the dimensions (width and length) of the garden that will maximize the area of the garden. What is the maximum area?

  f. What should be the dimensions of the garden if it is decided that the area enclosed should be 60 square feet?

10. A rectangular storage container with an open top and square base is to have a volume of 32 ft^3. Material for the base costs $6 per square foot and material for the sides cost $4 per square foot.

  a. Sketch a picture of the problem.

  b. If the width (and length) of the base is 2 feet, what is the area of the base? What is the cost of the material to make the base? What is the height of the container? What is the area of one side of the container? What is the area of all 4 sides of the container? What is the cost of the material for the 4 sides? What is the cost of the material for the entire container?

  c. If the width (and length) of the base is x feet, write an algebraic representation for:

    i. the area of the base.

    ii. the cost of the material to make the base.

    iii. the height of the container.

    iv. the area of one side of the container.

    v. the area of 4 sides of the container.

    vi. the cost of the material for the 4 sides.

    vii. the function C(x), that determines the total cost of the material for the entire container.

  d. Using the function C(x), graphically find the dimensions of the container, and the cost of the materials for the cheapest such container.