SUBSTITUTION (7.1) NAME_____

For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int (x^3+1)\,dx$ (3)

$= \boxed{\frac{x^4}{4} + x + C}$

$\int x^2(x^3+1)^4\,dx$ (1)

$u = x^3+1 \rightarrow \frac{du}{dx} = 3x^2$

$\rightarrow du = 3x^2\,dx$

$\rightarrow x^2\,dx = \frac{1}{3}\,du$

$= \int u^4 \cdot \left(\frac{1}{3}\,du\right)$

$= \frac{1}{15}u^5 + C = \boxed{\frac{1}{15}(x^3+1)^5 + C}$

$\int \sqrt{x^3+1}\,dx$ (4)

$\int (x^3+1)^2\,dx$ (2)

$= \int (x^6 + 2x^3 + 1)\,dx$

$= \boxed{\frac{x^7}{7} + \frac{x^4}{2} + x + C}$

B. $\int \sqrt{x}(1-x^2)\,dx$ (2)

$= \int (x^{1/2} - x^{5/2})\,dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{7/2}}{7/2} + C$

$= \boxed{\frac{2}{3}x^{3/2} - \frac{2}{7}x^{7/2} + C}$

$\int \sqrt{1-x^2}\,dx$ (4)

$\int \frac{1}{\sqrt{1-x^2}}\,dx$ (3)

$= \boxed{\arcsin(x) + C}$

$\int \frac{x\,dx}{\sqrt{1-x^2}}$

$u = 1-x^2 \rightarrow \frac{du}{dx} = -2x$

$\rightarrow du = -2x\,dx$

$\rightarrow x\,dx = -\frac{1}{2}\,du$

$= \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2}\,du\right)$

$= -\frac{1}{2}\int u^{-1/2}\,du$

$= -\frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + C = \boxed{-\sqrt{1-x^2} + C}$

C. $\int \cos^2 x \sin^3 x\,dx$ (1)

$= \int \cos^2(x) \cdot \sin(x) \cdot \sin^2(x)\,dx$

$= \int \cos^2(x) \cdot \sin(x) \cdot [1-\cos^2(x)]\,dx$

$= \int \sin(x)\cos^2(x)\,dx - \int \sin(x)\cos^4(x)\,dx$

$u = \cos(x) \rightarrow \frac{du}{dx} = -\sin(x) \rightarrow du = -\sin(x)\,dx$

$= \int -u^2\,du + \int u^4\,du$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C = \boxed{-\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C}$

$\int \sqrt{1-\cos^2 x}\,dx$ (2)

$= \int \sqrt{\sin^2(x)}\,dx$

$= \int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

$\int \frac{dx}{\cos^2 x}$ (3)

$= \int \sec^2(x)\,dx$

$= \boxed{\tan(x) + C}$

$\int \frac{dx}{\cos x \sqrt{\sin x}}$ (4)

D. $\int \tan x \sec x\,dx$ (3)

$= \boxed{\sec(x) + C}$

$\int \tan x \cos x\,dx$ (2)

$= \int \frac{\sin(x)}{\cos(x)} \cdot \cos(x)\,dx$

$= \int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

$\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$ (1)

$u = \tan(x) \rightarrow \frac{du}{dx} = \sec^2(x)$

$\rightarrow du = \sec^2(x)\,dx$

$= \int \frac{1}{\sqrt{u}}\,du$

$= \int u^{-1/2}\,du$

$= \frac{u^{1/2}}{1/2} + C = \boxed{2\sqrt{\tan(x)} + C}$

$\int \frac{dx}{\tan x + 1}$ (4)

E. $\int e^{-x^2}\,dx$ (4)

$\int \frac{e^x}{3+e^x}\,dx$ (1)

$u = 3+e^x \rightarrow \frac{du}{dx} = e^x$

$\rightarrow du = e^x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C = \boxed{\ln|3+e^x| + C}$

$\int (e^x + 3)\,dx$ (3)

$= \boxed{e^x + 3x + C}$

$\int \frac{\ln(e^{2x})}{x^2}\,dx$ (2)

$= \int \frac{2x}{x^2}\,dx$

$= 2\int \frac{1}{x}\,dx$

$= \boxed{2\ln|x| + C}$