STATISTICAL TESTS
Math 160 Section 3

ALL SAMPLES MUST BE SIMPLE RANDOM SAMPLES.

1 z Tests

1.1 z Test for a Population Mean

We estimate the population mean μ using the sample mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{Distribution: } N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Hypothesis Test To test $H_0 : \mu = \mu_0$, use **Test Statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \qquad \text{Distribution: } N(0,1)$$

Confidence Interval

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

where z^* is a critical value from the $N(0,1)$ distribution.
Guidelines for Use

- The population distribution should be close to normal unless n is large. This means the sample data must be symmetric with no outliers or skewness.
- We must know the population standard deviation, σ. This is unrealistic in practice.

1.2 z Test for a Population Proportion

We estimate the population proportion p using the **sample proportion**

$$\hat{p} = \frac{\#\text{ successes}}{n} \qquad \text{Distribution: } N\left(p, \sqrt{\frac{p(p-1)}{n}}\right)$$

$$\text{Standard Error: } \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

or the **plus-four proportion**

$$\tilde{p} = \frac{\#\text{ successes } + 2}{n+4} \qquad \text{Distribution: } N\left(p, \sqrt{\frac{p(p-1)}{n+4}}\right)$$

$$\text{Standard Error: } \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$$

Hypothesis Test To test $H_0 : p = p_0$, use the **Test Statistic**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \qquad \text{Distribution: } N(0,1)$$

- The sample size n must be large enough that np_0 and $n(1-p_0)$ are at least 10.

Large Sample Confidence Interval

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is a critical value from the $N(0,1)$ distribution.

- The number of successes and failures in the sample must both be at least 15.

Plus-Four Confidence Interval

$$\tilde{p} \pm z^* \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$$

where z^* is a critical value from the $N(0,1)$ distribution.

- The sample size n must be at least 15, and the confidence level must be at least 90%.

2 t Tests

2.1 One Sample t Test for a Population Mean

We estimate the population mean μ using the sample mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{Distribution: } N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$\text{Standard Error: } \frac{s}{\sqrt{n}}$$

Hypothesis Test To test $H_0 : \mu = \mu_0$, use the **Test Statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \text{Distribution: } t(n-1)$$

Confidence Interval

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t^* is a critical value from the $t(n-1)$ distribution.

Guidelines for Use

- Do not use the t-test if the sample data contains outliers.
- If $n < 15$, and the sample data is at all skewed, do not use the t-test.
- If $15 \leq n < 40$, and the sample data is very skewed, do not use the t-test.
- If $n \geq 40$, the t-test can be used even if the data if very skewed.

2.2 Two Sample t Test for a Difference of Population Means

We estimate the difference in means of two populations, $\mu_1 - \mu_2$, using the difference of sample means

$$\bar{x}_1 - \bar{x}_2 \qquad \text{Distribution: } N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \sigma_2^2 n_2}\right)$$

$$\text{Standard Error: } \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Hypothesis Test To test $H_0 : \mu_1 - \mu_2 = 0$, use the **Test Statistic**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad \text{Distribution: } t(df)$$

Confidence Interval

$$\bar{x} \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t^* is a critical value from the $t(df)$ distribution.

Degrees of Freedom A conservative estimate of the degrees of freedom is

$$\tilde{df} = \min\{n_1 - 1, n_2 - 1\}$$

The actual degrees of freedom are

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$$

Guidelines for Use

- We must have two samples, one each from two different populations, and the samples must be drawn independently.
- The samples **should** be equal in size and have similar distributions.
- If $n_1 + n_2 < 15$, and either set of sample data is at all skewed, do not use the two sample t-test.
- If $15 \leq n_1 + n_2 < 40$, and either set of sample data is very skewed, do not use the two sample t-test.
- If $n_1 + n_2 \geq 40$, the two sample t-test can be used even if the data is very skewed.

3 χ^2 Test

3.1 χ^2 Test for the Relationship Between Two Categorical Variables

To test the hypothesis H_0 : there is no relationship between categorical variables A and B, use the **Test Statistic**

$$\chi^2 = \sum \frac{(\text{EXPECTED} - \text{OBSERVED})^2}{\text{EXPECTED}} \qquad \text{Distribution: } \chi^2((r-1)(c-1))$$

where A has r labels, B has c labels, and for each entry in the table,

$$EXPECTED = \frac{(\text{total for row})(\text{total for column})}{\text{total for table}}$$

Guidelines for Use

- We have a two-way table that results from multiple SRSs, from different populations, with each individual classified according to a single categorical variable.
- OR we have a two-way table that results from a single SRS, with each individual classified according to two categorical variables.
- All expected cell counts are at least one.
- At least 80% of the expected cell counts are at least 5.

3.2 χ^2 Test for Goodness of Fit

To test $H_0 : p_1 = p_{10}, \ldots, p_k = p_{k0}$ (the hypothesis that the probabilities $p_1, \ldots, p_k$ have the values $p_{10}, \ldots, p_{k0}$) use the **Test Statistic**

$$\chi^2 = \sum \frac{(\text{count of outcome } i - np_{i0})^2}{np_{i0}} \qquad \text{Distribution: } \chi^2(k-1)$$