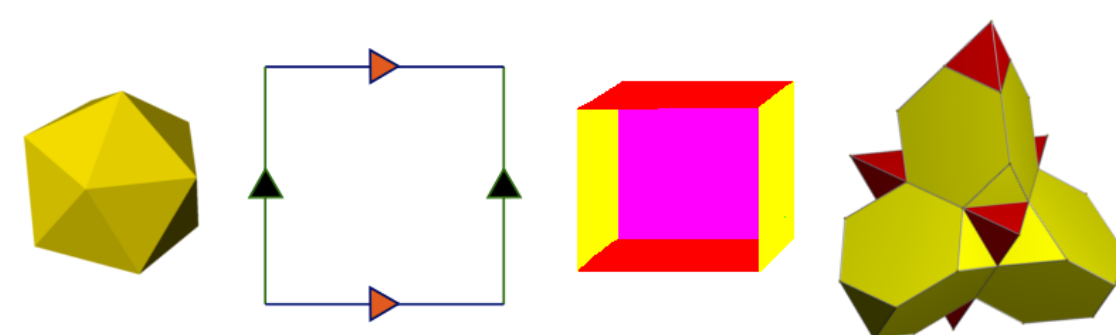


Introduction

In studying manifolds, it is desirable to be able to “step inside” of them and view them as a “native creature” would. By approximating smooth manifolds by piecewise flat manifolds that preserve topological information, we are able to use a source unfolding algorithm to create a visualization that reflects topological properties of the original manifold.

Piecewise Flat Manifolds

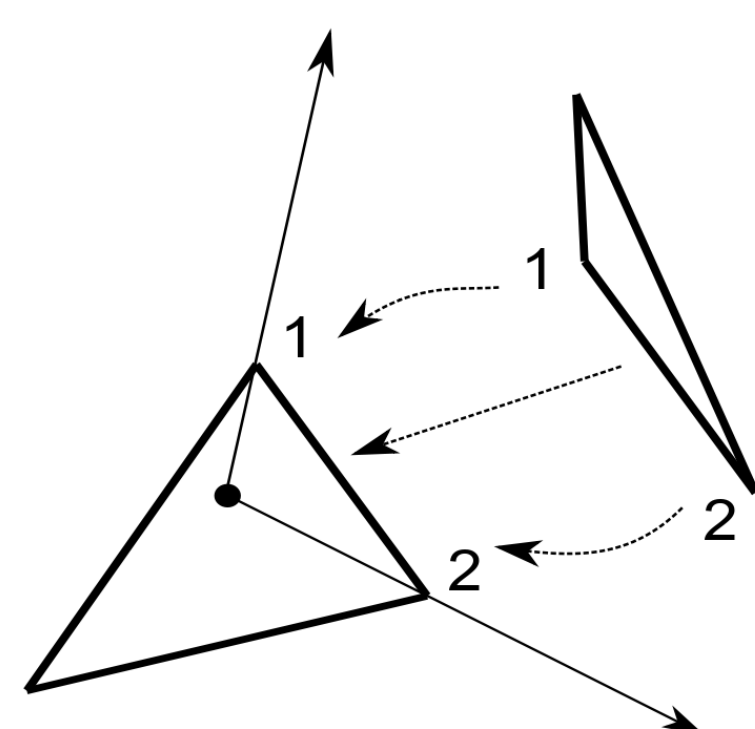
- An n -dimensional manifold is a space that is locally homeomorphic to \mathbb{R}^n .
- A **piecewise flat manifold** consists of some number of n -dimensional facets connected along their ridges.
 - Exactly two facets are connected at each ridge.
 - At lower dimensional boundaries facets must make a sphere.



- Each facet can be embedded in \mathbb{R}^n .
- Transition maps connecting facets along ridges are encoded using affine transformations.

Affine Transformations

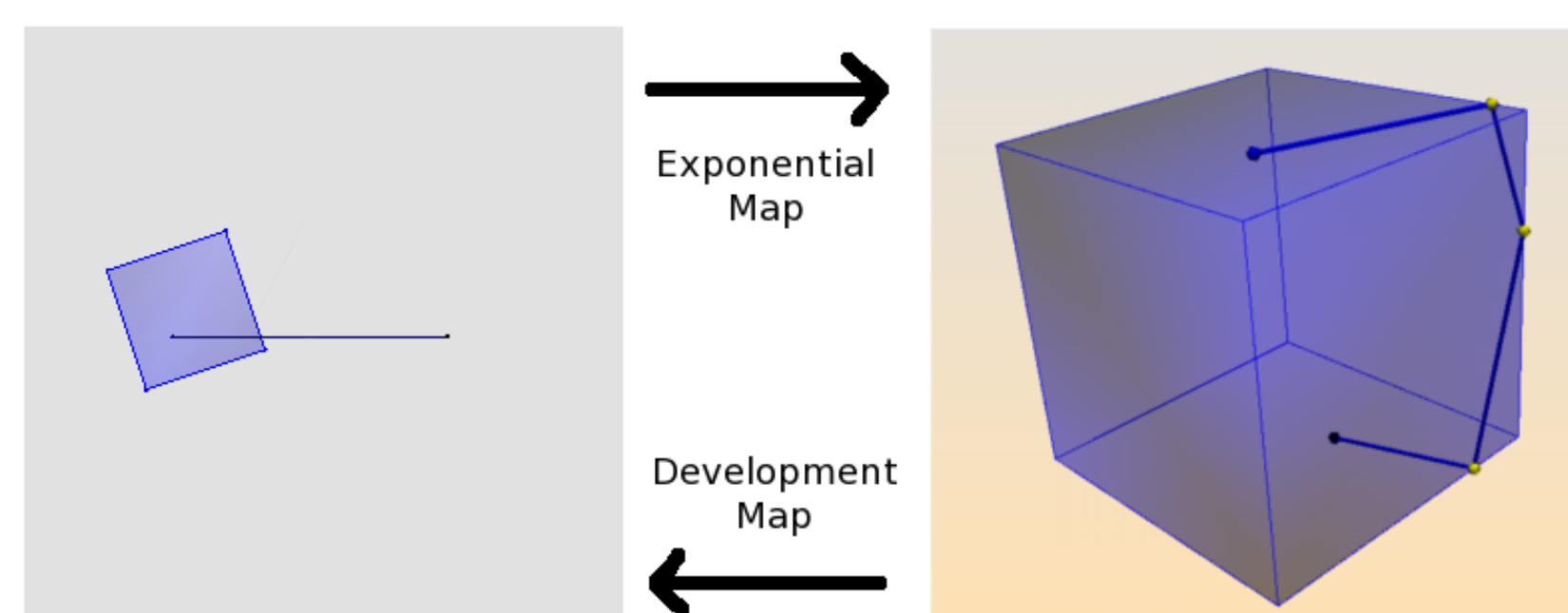
An **affine transformation** is a linear transformation followed by a translation, i.e., we can represent an affine transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ by the expression $T\mathbf{x} = A\mathbf{x} + \mathbf{b}$, where A is a linear transformation and \mathbf{b} is a vector.



Definitions

- Facet:** An n -facet is a finite intersection of closed half spaces in \mathbb{R}^n with non-zero volume.
- Ridge:** A codimension 1 facet.
- Geodesic:** A geodesic is a locally shortest path between two points.
- Tangent Space of a Point:** An n -dimensional plane that contains the facet upon which a point from the manifold lies.
- Visible:** A point defined by a vector in the tangent space is visible if the image of the vector under the exponential map contains no points on facets of codimension two or larger.
- Blocked Line of Sight:** A line of sight that passes through non-visible points.

Exponential Map and Development

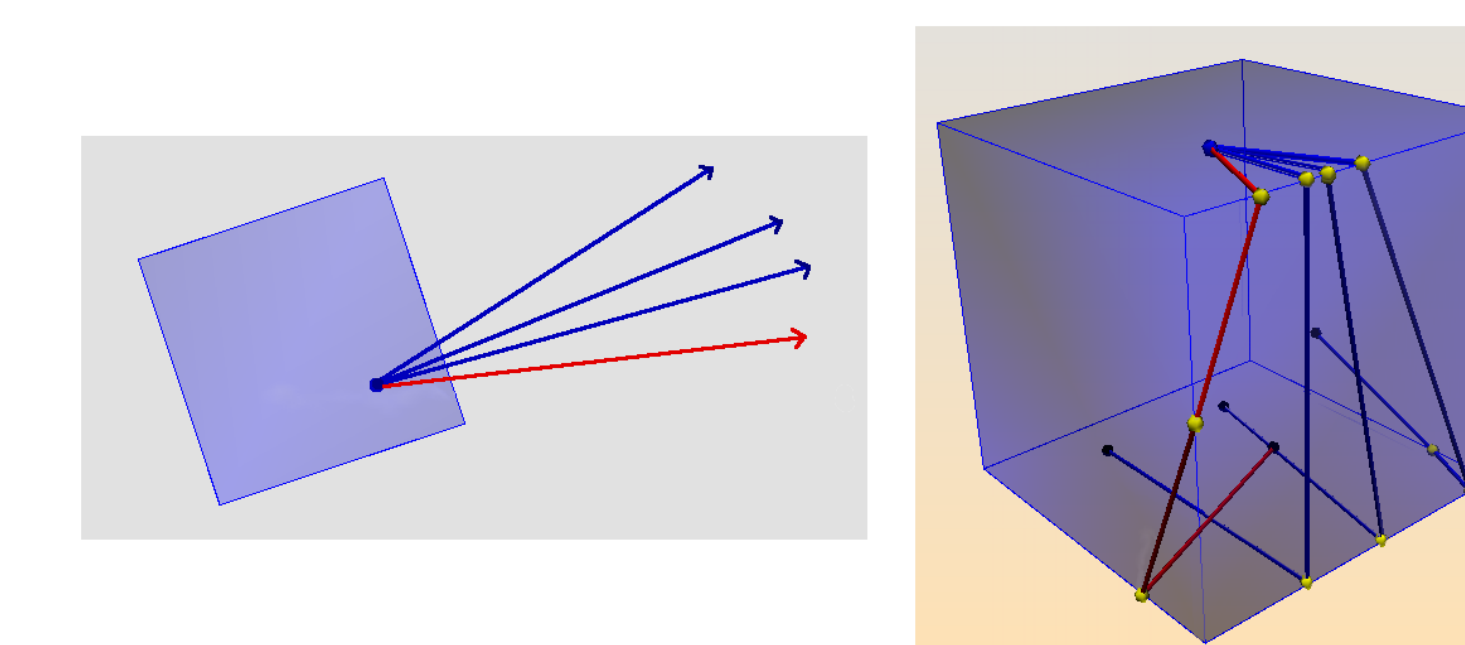


Given a point p on a manifold M and a vector v in its tangent space T , the **exponential map** returns the point on M determined by moving a distance $\|v\|$ along the geodesic through p in the initial direction of v for the distance equal to the magnitude of v . The point p is called a **source point**.

The inverse relation of this map, the **development**, carries points on M to (multiple) points in T .

We have constructed a **source unfolding map** that carries facets and parts of facets from M to T . The source unfolding is a model of the development relation.

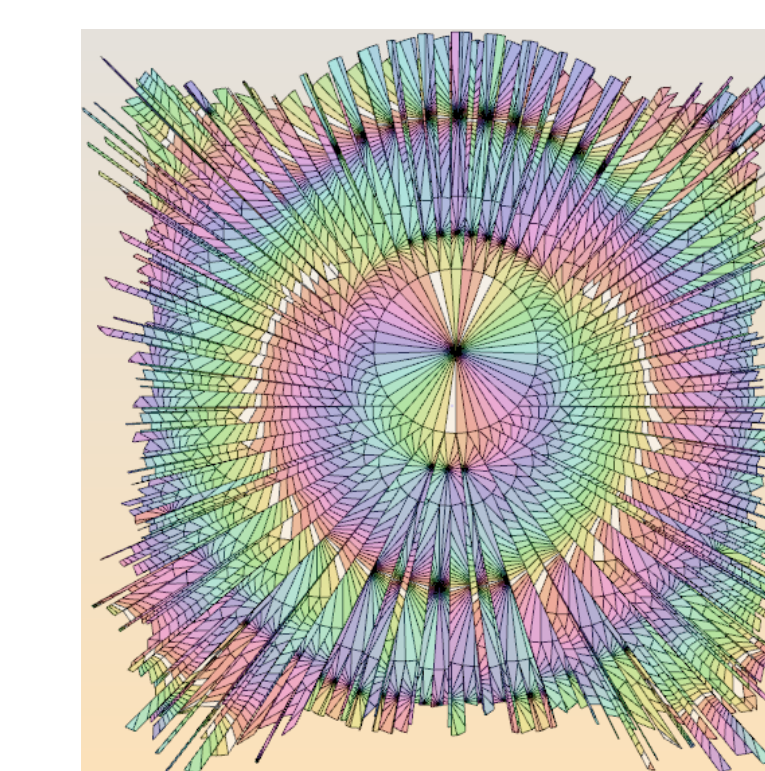
Frustums



A **frustum** determines part of the visual field in T . It can be defined either as a set of vectors in T that cross the same list of edges when scaled by a parameter $c > 0$, or as the intersection of half-spaces in T which intersect at the source point.

Results

- By implementing the source unfolding algorithm in JAVA, we were able to produce images depicting the visual field of 2-dimensional piecewise flat manifolds.



Source unfolding of a 40-faced pyramid.

- Theorem:** Blocked lines of sight are dense in the lines of sight emanating from any source point.

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