

Worksheet, Section 3.1

Solutions

A polynomial of **order** n is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

and the **leading term** is $a_n x^n$.

1. Consider the graph of a **monomial** $y = ax^n$. If a monomial is even-powered (even order n) then it will be bowl shaped. If a is positive, it will open up, and if a is negative, it will open down.

If the monomial is odd-powered, it will be positive on one side of the origin and negative on the other side.

(a) What are the conditions that determine specifically where the odd-powered monomial is positive and negative?

For an odd-powered monomial, if $a > 0$, then $y < 0$ when $x < 0$ and $y > 0$ when $x > 0$.

If $a < 0$, $y > 0$ when $x < 0$ and $y < 0$ when $x > 0$.

Graph a bunch of examples to see the pattern.

(b) Determine the end behavior (i.e. how the function behaves as $x \rightarrow \infty$ and $x \rightarrow -\infty$) of the following monomials.

1. $y = -4x^2$
As $x \rightarrow \infty$, $y \rightarrow -\infty$.
As $x \rightarrow -\infty$, $y \rightarrow -\infty$.

2. $y = \frac{1}{2}x^5$
 $x \rightarrow \infty$, $y \rightarrow \infty$.
 $x \rightarrow -\infty$, $y \rightarrow -\infty$.

3. $y = -x^3$
 $x \rightarrow \infty$, $y \rightarrow -\infty$.
 $x \rightarrow -\infty$, $y \rightarrow \infty$.

4. $y = -8x^{12}$
 $x \rightarrow \infty$, $y \rightarrow -\infty$.
 $x \rightarrow -\infty$, $y \rightarrow -\infty$.

2. The end behavior of a polynomial is determined by its leading term. Describe the end behavior of the following polynomials.

1. $y = 6x^3 - 5x^2 + 4x - 3$ Leading term: $y = 6x^3$.
So as $x \rightarrow \infty$, $y \rightarrow \infty$,
 $x \rightarrow -\infty$, $y \rightarrow -\infty$.

2. $y = 2(x-3)^2(x+4)$ Leading term: $y = 2x^3$.
So as $x \rightarrow \infty$, $y \rightarrow \infty$,
 $x \rightarrow -\infty$, $y \rightarrow -\infty$.