Worksheet, Section 3.1

A polynomial of **order** n is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0,$$

and the **leading term** is $a_n x^n$.

1. Consider the graph of a **monomial** $y = ax^n$. If a monomial is even-powered (even order n) then it will be bowl shaped. If a is positive, it will open up, and if a is negative, it will open down.

If the monomial is odd-powered, it will be positive on one side of the origin and negative on the other side.

(a) What are the conditions that determine specifically where the odd-powered monomial is positive and negative?

(b) Determine the end behavior (i.e. how the function behaves as $x \to \infty$ and $x \to -\infty$) of the following monomials.

1. $y = -4x^2$

2. $y = \frac{1}{2}x^5$

3. $y = -x^3$

4. $y = -8x^{12}$

2. The end behavior of a polynomial is determined by its leading term. Describe the end behavior of the following polynomials.

1. $y = 6x^3 - 5x^2 + 4x - 3$

2. $y = 2(x-3)^2(x+4)$

3. Find all zeros of the following polynomials. Remember that for any polynomial $P(x)$, c is a zero (or $(c, 0)$ is an x-intercept) if $(x - c)$ is a factor of $P(x)$.

1. $P(x) = x(x^2 - 5x + 4)$

2. $Q(x) = x^3 - x^2 - 7x + 7$ Hint: graph it on your calculator to get started

3. $R(x) = (6x - 5)(x + 2)(4x + 9)$

4. $S(x) = x^4 - 3x^3 + 2x^2$

4. The graphs of every polynomial share a number of properties. For example, they are all continuous, smooth (i.e. no cusps or sharp edges), and they all diverge as $x \to \pm\infty$. Find 3 examples of polynomials with different orders that do not have any x-intercepts.

5. What class of orders of polynomials must always have at least one zero? Explain precisely why this must be true.