Worksheet, Section 2.2
Solutions

1. Find all intercepts and the vertex of the following functions:

(a) $f(x) = (x-4)^2 - 1$

- Vertex: $(4, -1)$
- y-intercept: $f(0) = (0-4)^2 - 1 = 15$, so $(0, 15)$.
- x-intercept: set $0 = (x-4)^2 - 1$ so $x = 4 \pm \sqrt{1}$, or $x = 3, 5$

(b) $g(x) = 3x^2 - 6x - 9$

- Vertex: $x = -b/(2a) = 6/(2 \cdot 3) = 1$, so $y = g(1) = 3 - 6 - 9 = -12$. Then vertex $= (1, -12)$
- y-intercept: $g(0) = -9$, so $(0, -9)$.
- x-intercept: Using quadratic formula: $x = (6 \pm \sqrt{36 + 4 \cdot 3 \cdot 9})/(2 \cdot 3) = 1 \pm 2$. Or we can factor $g(x) = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$.

(c) $h(x) = -x^2 - 3x + 4$

- Vertex: $x = \dfrac{-(-3)}{2(-1)} = -1.5$, $y = h(1.5) = 6.25$.
- y-intercept: $h(0) = 4$
- x-intercept: $h(x) = -(x+4)(x-1)$, so $x = -4, 1$.

(d) $p(x) = -(x+1)^2 + 4$

- Vertex: $(-1, 4)$
- y-intercept: $h(0) = -1 + 4 = 3$
- x-intercept: set $0 = -(x+1)^2 + 4$ so $x = -1 \pm \sqrt{4}$, or $x = -3, 1$

2. For each of the quadratic functions in question 1, if the function is in general form put it in standard form, and vice versa.

(a) $f(x) = (x-4)^2 - 1 = x^2 - 8x + 16 - 1 = x^2 - 8x + 15$

(b) $g(x) = 3(x^2 - 2x) - 9 = 3\left((x-1)^2 - 1\right) - 9 = 3(x-1)^2 - 12$

(c) $h(x) = -(x^2 + 3) + 4 = -(x + \frac{3}{2})^2 + 4 - 2.25 = -(x + \frac{3}{2})^2 + 1.75$

(d) $p(x) = -(x^2 + 2x + 1) + 4 = -x^2 - 2x + 3$

3. Two holding pens are to be constructed with 800 feet of fencing. What is the maximum total area of the holding pens?

$A = w \cdot l$, and total fencing $= 800 = 3w + 2l$.

Then $l = \dfrac{800 - 3w}{2}$, so $A(w) = w\left(\dfrac{800 - 3w}{2}\right)$, which is a quadratic. We can find the maximum of $A(w)$, by simplifying $A(w)$ then using the formula for the vertex:

$$A(w) = w\left(\frac{800 - 3w}{2}\right) = -\frac{3}{2}w^2 + 400w$$

$$w_{max} = \frac{-400}{-3} = \frac{400}{3}$$

Then the maximum area is given by

$$A\left(\frac{400}{3}\right) = -\frac{3}{2}\left(\frac{400}{3}\right)^2 + 400\left(\frac{400}{3}\right) \approx 26,666.7\,\text{ft}^2$$

4. We are landcaping a rectangular yard. The total area of the yard is 40 feet wide and 60 feet long. There is to be a rectangular lawn in the middle of the yard with a planting strip of constant width around the side. The lawn is 1344 square feet. What are the dimensions of the lawn? How wide is the strip? First: draw a picture!

The width and length of the lawn are $40 - 2x$ and $60 - 2x$, where x is the width of the planting strip. Then

$$(40 - 2x)(60 - 2x) = 1344$$

$$2400 - 200x + 4x^2 = 1344$$

$$1056 - 200x + 4x^2 = 0$$

$$264 - 50x + x^2 = 0$$

and we solve for x using the quadratic formula to get $x = 6$ or $x = 44$. $x = 6$ is the reasonable answer for the width of the strip. Then the dimensions of the lawn are 28×48 square feet.