

# Automated Neuronal Reconstruction: Current Challenges and Proposed Quantification

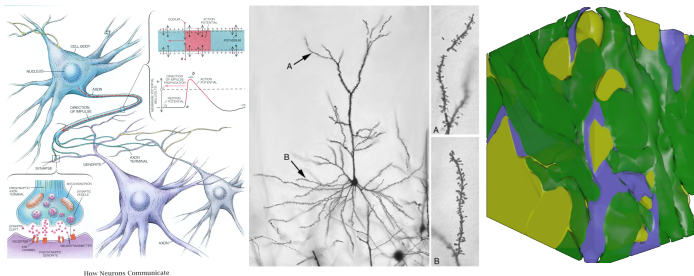
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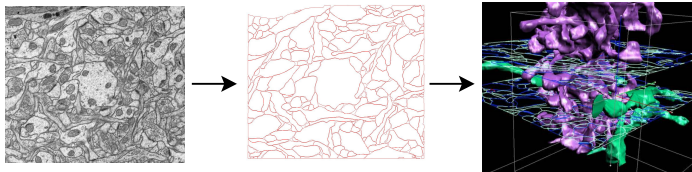
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- 2 Prior Work and Background
- 3 Current Challenges and Proposed Measurements

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# Motivation: Neuronal Modeling



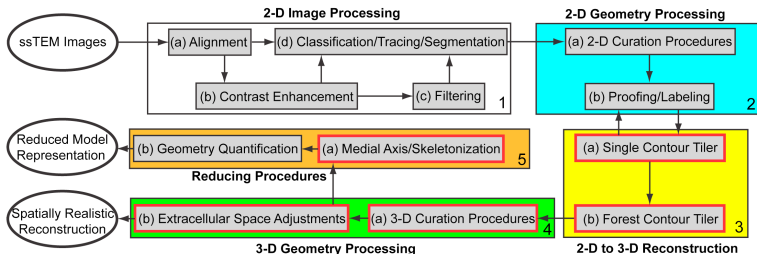
Neurons in vivo are packed very densely and have many small geometric features known to affect voltage decay.



Neuron length  $\sim 100 \mu\text{m}$ ; Neuropil dataset  $\sim 2 (\mu\text{m})^3$ ; in-plane resolution  $\sim 5\text{-}10 \text{ nm}$

# Pipeline Overview

The Automated Neuronal Reconstruction project is a vast undertaking. In this talk, we will focus only on Step 3: 2-D to 3-D Reconstruction.

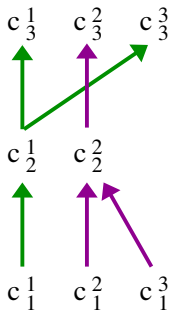
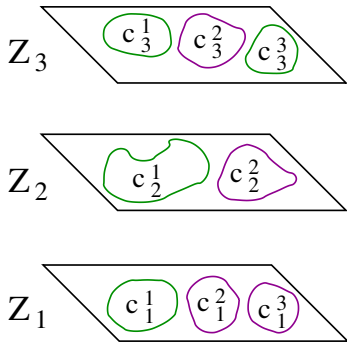


At the end of the talk we will discuss the questions we wish to answer with a complete, curated, spatially realistic 3-D reconstruction.

# Formal Problem Statement

## Input (for a $K$ component forest):

- 1  $M$  horizontal planes  $Z_1, \dots, Z_M$ . ( $Z_m$  given by  $z = z_m$ )
- 2  $K$  functions  $g_m^1, \dots, g_m^K : \mathbb{R}^2 \rightarrow \mathbb{R}$  such that  $\bigcup_{k=1}^K \{g_m^k = 0\}$  is a 1-manifold.
- 3 Contours  $\{c_m^k\}$  of the set  $\{g_m^k = 0\}$ .
- 4 An acyclic directed graph  $G$  with vertices  $\{c_m^k\}$ , indicating connectivity.



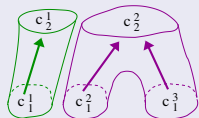
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## Output $K$ functions $h_1, \dots, h_K : \mathbb{R}^3 \rightarrow \mathbb{R}$ such that:

- 1 Each  $h_k$  restricts to  $g_m^k$  on  $Z_m$ , i.e.  
 $h_k(x, y, z_m) \equiv g_m^k(x, y)$ .
- 2 Each surface  $h_k(x, y, z) = 0$  is a compact, connected, smooth 2-manifold with local connectivity corresponding to the graph  $G$ .
- 3 The  $K$  component surface  $\prod_{k=1}^K h_k(x, y, z) = 0$  is a 2-manifold, i.e. the component surfaces do not intersect.



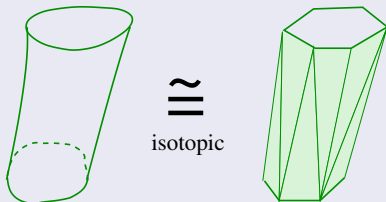
# Simplifying Assumptions

## Assumptions for Reduction to Meshing:

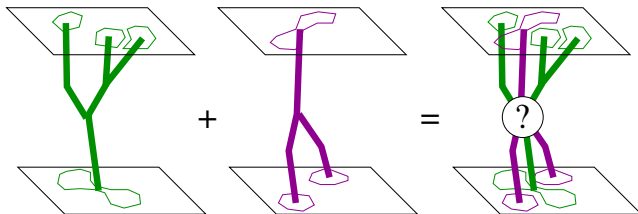
- 1 Contours are simple polygons and can be refined if necessary.



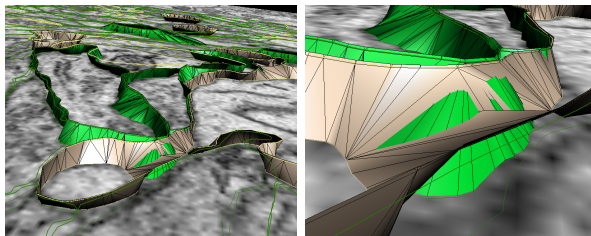
- 2 A mesh of the polygonal contours satisfying the output properties is isotopic to a smooth solution.



# The Multi-Component Difficulty



Independent solutions to the reconstruction problem for each component may produce topological or geometrical inaccuracies when aggregated.



- 1 Motivation and Problem Statement
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# Selected Prior Work on Single Component Problem

FUCHS, KEDEM AND USELTON *Optimal surface reconstruction from planar contours* Communications of the ACM 20:10, 1977.

- Seminal work in reconstruction from polygonal contours.

MEYERS, SKINNER AND SLOAN *Surfaces from contours* ACM Transactions on Graphics 11:3, 1992.

- Identified subproblems of correspondence, tiling, and branching.

BAREQUET AND SHARIR *Piecewise-linear interpolation between polygonal slices* Symposium on Computational Geometry 1994.

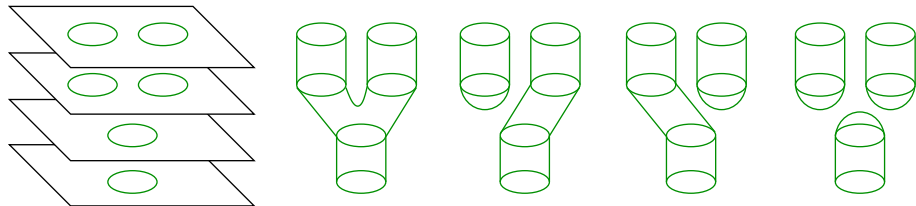
- Developed an algorithm for CT, MRI, and other medical applications.

BAJAJ, COYLE AND LIN *Arbitrary topology shape reconstruction from planar cross sections* Graphic Models and Image Processing 58:6, 1996.

- Expanded this algorithm by providing topological guarantees on the output.
- We use this method for our approach.

# The Correspondence Problem

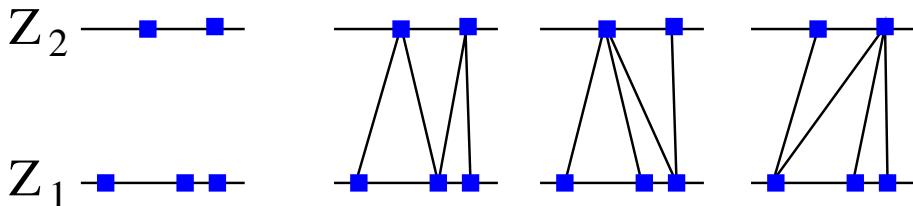
How should contours on adjacent slices connect?



Remark: This is resolved by the connectivity graph  $G$  in our case.

# The Tiling Problem

How should corresponding contours be tiled?



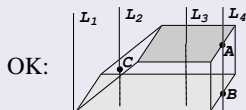
Definition: A **slice chord** is an edge connecting vertices on adjacent slices. A **tiling triangle** is formed by two slice chords and a contour edge.

*What are the criteria of a “good” tiling?*

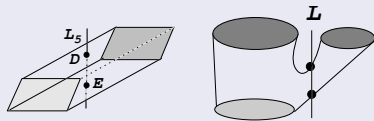
# The Tiling Problem

## Desired Tiling Criteria

- 1 The reconstructed surface forms a piecewise closed surface of polyhedra.
- 2 Any vertical line segment between two adjacent slices intersecting the reconstructed surface does so at exactly one point or along exactly one line segment.



not OK:

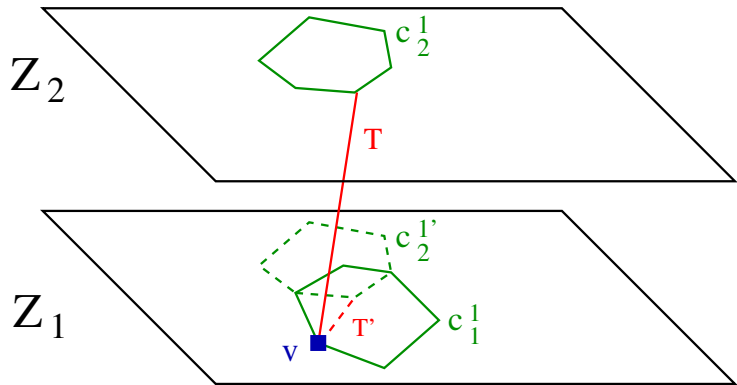


- 3 Resampling of the reconstructed surface on any slice reproduces the original contours.

Remark: The last criterion implies that aside from contour refinement, any edges or vertices added to the mesh must lie outside the  $Z_i$  planes.

# The Tiling Problem Resolved

- Let  $v$  be a vertex in contour  $c_1^1 \subset Z_1$  corresponding to  $c_2^1 \subset Z_2$ .
- Let  $T$  be a slice chord from  $v$  to  $c_2^1$ .
- Let the "prime" notation denote vertical projection to  $Z_1$ .

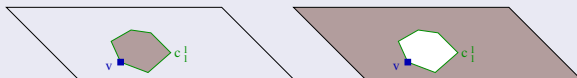


# The Tiling Problem Resolved

## Theorem [Bajaj, Coyle, Lin 1996]:

If a tiling satisfies the three criteria, the following hold:

i)  $T'$  lies in exactly one of these regions:



ii) If  $v \notin c_2^{1'}$  then  $T'$  lies in exactly one of these regions:



iii) If  $v \in c_2^{1'}$  then  $T'$  lies in exactly one of these regions:



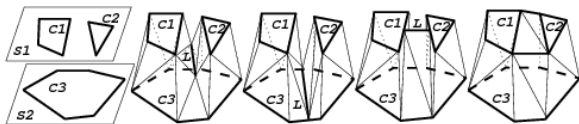
# The Tiling Problem Resolved

## Tiling Algorithm [*Bajaj, Coyle, Lin 1996*]:

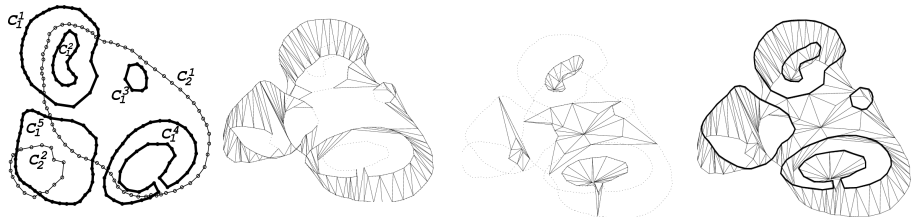
- 1 For each vertex  $v \in \{c_1^j\}$ , make a list of all the slice chords that could be formed to a vertex of  $\{c_2^k\}$  (based on the resolution of the correspondence problem).
- 2 Select the shortest length chord from this list which satisfies the results of the Theorem.
- 3 If no chord from the list satisfies the theorem, tag the vertex as “untiled.”
- 4 Collect boundaries of untiled regions for subsequent meshing when resolving the branching problem.

# The Branching Problem

How should tiling be done when a contour in  $Z_1$  corresponds to more than one contour in  $Z_2$ ?



To ensure the criteria are satisfied, we add vertices to a plane half way between  $Z_1$  and  $Z_2$  and then mesh.



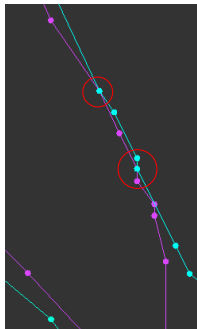
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# Overview of Challenges

We will address three of the challenges currently being tackled to adapt the existing software CONTOURTILER to the needs of ssTEM data.

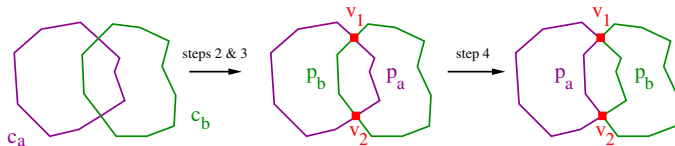
- 1 2D contour overlap and proximity
- 2 Bubble versus pocket creation
- 3 Geometry problems with enclosed contours

# 2D Contour Overlap and Proximity

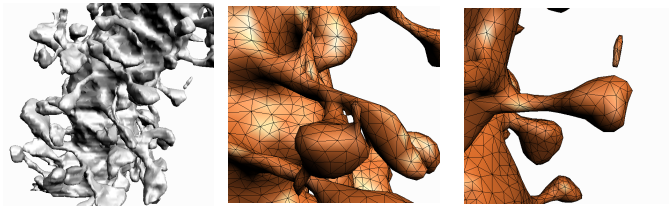
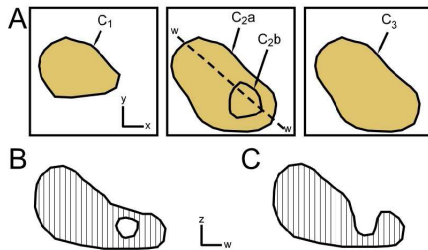


**Input:** Collection of  $n$  piecewise linear contours on a plane.

- 1 Read in a pair of contours  $c_a$  and  $c_b$ .
- 2 Take the symmetric difference of  $c_a$  and  $c_b$ . The output is a  $k$ -component polygon  $P$  where  $k \geq 0$ .
- 3 The boundary of  $P$  divides into two connected chains of line segments,  $p_a$  belonging to  $c_a$  and  $p_b$  belonging to  $c_b$ . The two chains intersect at vertices  $v_1$  and  $v_2$  and bound the polygon  $P$ . Collect pointers to  $p_a$ ,  $p_b$ ,  $v_1$ , and  $v_2$ .
- 4 Reassign the edges of  $p_a$  to  $c_b$  and the edges of  $p_b$  to  $c_a$  in accordance with the oriented naming of  $c_a$  and  $c_b$ . The contours now touch at  $v_1$  and  $v_2$  and do not intersect otherwise.

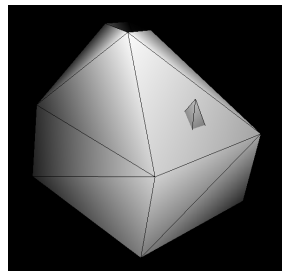
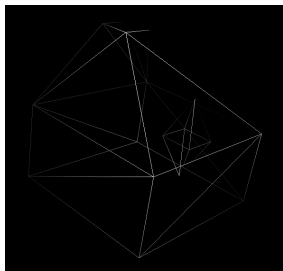
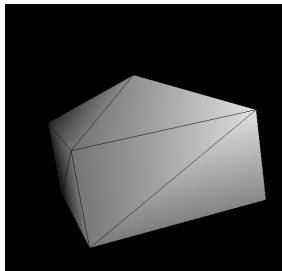
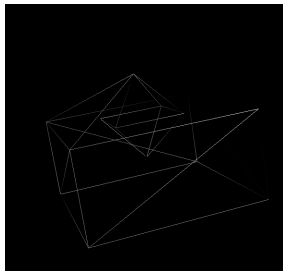


# Bubble Versus Pocket Creation



Top figure is by Justin Kinney, Salk Institute.

# Geometry problems with enclosed contours



# Proposed Quantification Questions

- 1 What fraction of the neuropil volume is occupied by dendrites? axons? glial cells? extracellular space?
- 2 What fraction of the surface area of dendrites are synapses? What fraction is in close proximity with glial cells? What do these results suggest for a contact region spatial distribution function?
- 3 What is the average number of spines per dendrite? What is the average number of glial-dendrite contact regions per unit length along dendrite?
- 4 How can the growth phenomena of cells in this region of the brain be characterized? Can we measure the winding or twisting of the various processes to help understand how growth falls on a scale of random to directed?

# Acknowledgements



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