Basis Functions for Serendipity Finite Element Methods

Andrew Gillette

Department of Mathematics
University of California, San Diego

http://ccom.ucsd.edu/∼agillette/

14th International Conference Approximation Theory

What is a serendipity finite element method?

Goal: Efficient, accurate approximation of the solution to a PDE over $\Omega \subset \mathbb{R}^n$.

Standard $O(h^r)$ **tensor product** finite element method in $\mathbb{R}^n$:

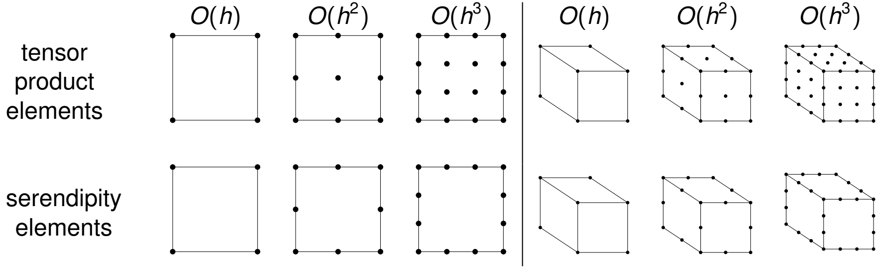
$\rightarrow$ Mesh Ω by n-dimensional cubes of side length h.

$\rightarrow$ Set up a linear system involving $(r+1)^n$ degrees of freedom (DoFs) per cube.

$\rightarrow$ For unknown continuous solution u and computed discrete approximation u_h:

$$\underbrace{||u - u_h||_{H^1(\Omega)}}_{\text{approximation error}} \leq \underbrace{C\, h^r\, |u|_{H^{r+1}(\Omega)}}_{\text{optimal error bound}}, \quad \forall u \in H^{r+1}(\Omega).$$

A $O(h^r)$ **serendipity** FEM converges at the **same rate** with **fewer DoFs** per element:

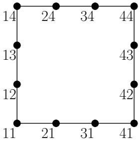


Example: For $O(h^3)$, $d = 3$, 50% fewer DoFs $\rightarrow$ $\boxed{\approx 50\% \text{ smaller linear system}}$

What is a geometric decomposition?

A **geometric decomposition** for a finite element space is an explicit correspondence:

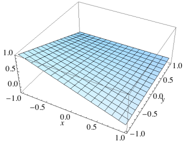
$$\{1, x, y, x^2, y^2, xy, x^3, y^3, x^2y, xy^2, x^3y, xy^3\} \longleftrightarrow \{\vartheta_{11}, \vartheta_{14}, \vartheta_{41}, \vartheta_{44}, \vartheta_{12}, \vartheta_{13}, \vartheta_{42}, \vartheta_{43}, \vartheta_{21}, \vartheta_{31}, \vartheta_{24}, \vartheta_{34}\} \longleftrightarrow$$



monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

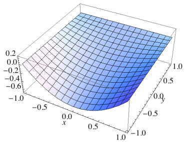
$\rightarrow$ Previously known basis functions employ Legendre polynomials

$\rightarrow$ These functions bear no symmetrical correspondence to the domain points and hence are not useful for isogeometric analysis.



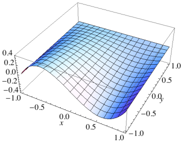
$\frac{1}{4}(x-1)(y-1)$

vertex



$-\frac{1}{4}\sqrt{\frac{3}{2}}\left(x^2-1\right)(y-1)$

edge (quadratic)



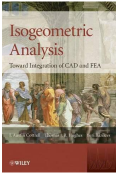
$-\frac{1}{4}\sqrt{\frac{5}{2}}x\left(x^2-1\right)(y-1)$

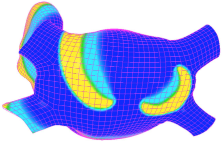
edge (cubic)

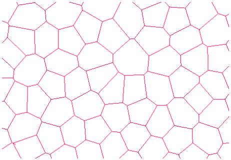
SZABÓ AND I. BABUŠKA *Finite element analysis*, Wiley Interscience, 1991.

Motivations and Related Topics

Goal: Construct geometric decompositions of serendipity spaces using linear combinations of standard tensor product functions. **Focus:** Cubic Hermites.







- **Isogeometric analysis:** Finding basis functions suitable for both domain description and PDE approximation avoids the expensive computational bottleneck of re-meshing.

  COTTRELL, HUGHES, BAZILEVS *Isogeometric Analysis: Toward Integration of CAD and FEA*, Wiley, 2009.

- **Modern mathematics:** Finite Element Exterior Calculus, Discrete Exterior Calculus, Virtual Element Methods. . .

  ARNOLD, AWANOU *The serendipity family of finite elements*, Found. Comp. Math, 2011.
  DA VEIGA, BREZZI, CANGIANI, MANZINI, RUSSO *Basic Principles of Virtual Element Methods*, M3AS, 2013.

- **Flexible Domain Meshing:** Serendipity type elements for Voronoi meshes provide computational benefits without need of tensor product structure.

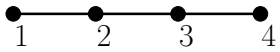
  RAND, G., BAJAJ *Quadratic Serendipity Finite Elements on Polygons Using Generalized Barycentric Coordinates*, Mathematics of Computation, in press.

Table of Contents

1. The Cubic Case: Hermite Functions, Serendipity Spaces
2. Geometric Decompsitions of Cubic Serendipity Spaces
3. Applications and Future Directions

Outline

1. The Cubic Case: Hermite Functions, Serendipity Spaces
2. Geometric Decompsitions of Cubic Serendipity Spaces
3. Applications and Future Directions

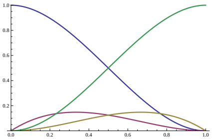
$\{1, x, x^2, x^3\}$ $\longleftrightarrow$ $\{\psi_1, \psi_2, \psi_3, \psi_4\}$ $\longleftrightarrow$ 

approxim. *geometry*

monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Cubic Hermite Basis on $[0,1]$

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} := \begin{bmatrix} 1-3x^2+2x^3 \\ x-2x^2+x^3 \\ x^2-x^3 \\ 3x^2-2x^3\end{bmatrix}$$

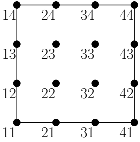


Approximation: $x^r = \sum_{i=1}^{4} \varepsilon_{r,i}\psi_i$, for $r = 0,1,2,3$, where $[\varepsilon_{r,i}] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -3 & 1 \end{pmatrix}$

Geometry: $u = u(0)\psi_1 + u'(0)\psi_2 - u'(1)\psi_3 + u(1)\psi_4, \qquad \forall u \in \underbrace{\mathcal{P}_3([0,1])}_{\text{cubic polynomials}}$

Cubic Hermite Geometric Decomposition: 2D

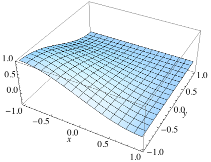
$$\underbrace{\left\{ \begin{array}{c} x^r y^s \\ 0 \leq r, s \leq 3 \end{array} \right\}}_{\mathcal{Q}_3([0,1]^2)} \quad \longleftrightarrow \quad \left\{ \begin{array}{c} \psi_i(x)\psi_j(y) \\ 1 \leq i, j \leq 4 \end{array} \right\} \quad \longleftrightarrow$$



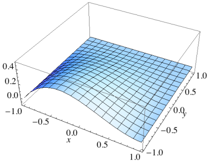
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \varepsilon_{r,i}\varepsilon_{s,j}\psi_{ij}$, for $0 \leq r, s \leq 3$, $\varepsilon_{r,i}$ as in 1D.

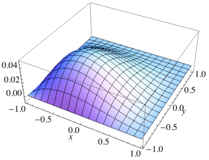
Geometry:



ψ_{11}



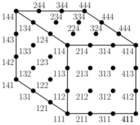
ψ_{21}



ψ_{22}

$$u = u|_{(0,0)}\psi_{11} + \partial_x u|_{(0,0)}\psi_{21} + \partial_y u|_{(0,0)}\psi_{12} + \partial_x\partial_y u|_{(0,0)}\psi_{22} + \cdots, \qquad \forall u \in \mathcal{Q}_3([0,1]^2)$$

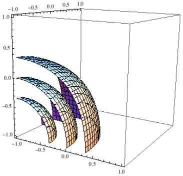
$$\underbrace{\left\{ \begin{array}{c} x^r y^s z^t \\ 0 \leq r,s,t \leq 3 \end{array} \right\}}_{\mathcal{Q}_3([0,1]^3)} \longleftrightarrow \begin{array}{c} \psi_i(x)\psi_j(y)\psi_k(z) \\ 1 \leq i,j,k \leq 4 \end{array} \longleftrightarrow$$



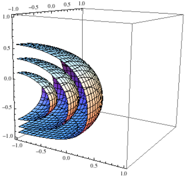
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s z^t = \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4} \varepsilon_{r,i}\varepsilon_{s,j}\varepsilon_{t,k}\psi_{ijk}$, for $0 \leq r,s,t \leq 3$, $\varepsilon_{r,i}$ as in 1D.

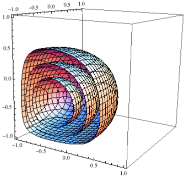
Geometry: Contours of level sets of the basis functions:



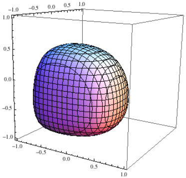
ψ_{111}



ψ_{112}



ψ_{212}



ψ_{222}

Two families of finite elements on cubical meshes

$\mathcal{Q}_r^- \Lambda^k([0,1]^n) \longrightarrow$ tensor product spaces ($\leq$ degree r in each variable)

early work: RAVIART, THOMAS 1976, NEDELEC 1980
more recently: ARNOLD, BOFFI, BONIZZONI arXiv:1212.6559, 2012

$\mathcal{S}_r \Lambda^k([0,1]^n) \longrightarrow$ serendipity finite element spaces (superlinear degree r)

early work: STRANG, FIX *An analysis of the finite element method* 1973
more recently: ARNOLD, AWANOU FoCM 11:3, 2011, and arXiv:1204.2595, 2012.

The **superlinear** degree of a polynomial ignores linearly-appearing variables.

$$n=2: \quad \{\overbrace{\underbrace{1, x, y, x^2, y^2, xy, x^3, y^3, x^2y, xy^2, x^3y, xy^3}_{\mathcal{S}_3\Lambda^0([0,1]^2)\ \text{(dim=12)}}, x^2y^2, x^3y^2, x^2y^3, x^3y^3}^{\mathcal{Q}_3\Lambda^0([0,1]^2)\ \text{(dim=16)}}\}$$

$$n=3: \quad \{\overbrace{\underbrace{1, \ldots, xyz, \ \ x^3y, x^3z, y^3z, \ldots, x^3yz, xy^3z, xyz^3}_{\mathcal{S}_3\Lambda^0([0,1]^3)\ \text{(dim=32)}}, x^3y^2, \ldots, x^3y^3z^3}^{\mathcal{Q}_3\Lambda^0([0,1]^3)\ \text{(dim=64)}}\}$$

$\mathcal{Q}_r^- \Lambda^k$ and $\mathcal{S}_r \Lambda^k$ and have the **same** key mathematical properties needed for FEEC
(degree, inclusion, trace, subcomplex, unisolvence, commuting projections)
but for fixed $k \geq 0$, $r, n \geq 2$ the serendipity spaces have **fewer** degrees of freedom

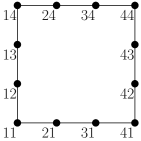
Outline

1. The Cubic Case: Hermite Functions, Serendipity Spaces
2. Geometric Decompsitions of Cubic Serendipity Spaces
3. Applications and Future Directions

Cubic Hermite Serendipity Geom. Decomp: 2D

Theorem [G, 2012]: A Hermite-like geometric decomposition of $\mathcal{S}_3([0,1]^2)$ exists.

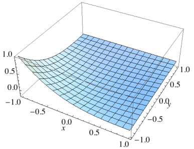
$$\underbrace{\left\{ \begin{array}{c} x^r y^s \\ \text{sldeg} \leq 3 \end{array} \right\}}_{\mathcal{S}_3([0,1]^2)} \longleftrightarrow \left\{ \begin{array}{c} \vartheta_{\ell m} \\ \text{(to be defined)} \end{array} \right\} \longleftrightarrow$$



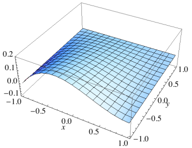
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s = \sum_{\ell m} \varepsilon_{r,i} \varepsilon_{s,j} \vartheta_{\ell m}$, for superlinear degree$(x^r y^s) \leq 3$

Geometry:



ϑ_{11}



ϑ_{21}

$$\begin{aligned} u &= u|_{(0,0)}\vartheta_{11} \\ &\quad + \partial_x u|_{(0,0)}\vartheta_{21} \\ &\quad + \partial_y u|_{(0,0)}\vartheta_{12} \\ &\quad + \cdots \end{aligned}$$

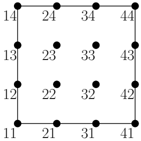
$\forall u \in \mathcal{S}_3([0,1]^2)$

Cubic Hermite Serendipity Geom. Decomp: 2D

Proof Overview:

1 Fix index sets and basis orderings based on domain points:

$$\begin{aligned} V &= \text{vertices } (11, 14, \ldots) \\ E &= \text{edges } (12, 13, \ldots) \\ D &= \text{interior } (22, 23, \ldots) \end{aligned} \quad \longleftrightarrow$$



$$\begin{aligned} [\vartheta_{\ell m}] &:= [\, \vartheta_{11}, \vartheta_{14}, \vartheta_{41}, \vartheta_{44}, \quad \vartheta_{12}, \vartheta_{13}, \vartheta_{42}, \vartheta_{43}, \vartheta_{21}, \vartheta_{31}, \vartheta_{24}, \vartheta_{34} \,], \\ [\psi_{ij}] &:= [\, \underbrace{\psi_{11}, \psi_{14}, \psi_{41}, \psi_{44},}_{\text{indices in } V} \; \underbrace{\psi_{12}, \psi_{13}, \psi_{42}, \psi_{43}, \psi_{21}, \psi_{31}, \psi_{24}, \psi_{34},}_{\text{indices in } E} \; \underbrace{\psi_{22}, \psi_{23}, \psi_{32}, \psi_{33}}_{\text{indices in } D} \,] \end{aligned}$$

2 Define a 12×16 matrix $\mathbb{H}$ with entries $h_{ij}^{\ell m}$ so that $\ell m \in V \cup E$, $ij \in V \cup E \cup D$.

3 Define the serendipity basis functions $\vartheta_{\ell m}$ via

$$\boxed{[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}]}$$

and show that the approximation and geometry properties hold.

Cubic Hermite Serendipity Geom. Decomp: 2D

Proof Details:

$\boxed{\mathbf{2}}$ Define a 12×16 matrix $\mathbb{H}$ with entries $h_{ij}^{\ell m}$ so that $\ell m \in V \cup E$, $ij \in V \cup E \cup D$.

$$\mathbb{H} := \left[\begin{array}{c|rrrr} & -1 & 1 & 1 & -1 \\ & 1 & -1 & -1 & 1 \\ & 1 & -1 & -1 & 1 \\ & -1 & 1 & 1 & -1 \\ & -1 & 0 & 1 & 0 \\ \mathbb{I} & 0 & -1 & 0 & 1 \\ \text{(12}x\text{12 identity matrix)} & 1 & 0 & -1 & 0 \\ & 0 & 1 & 0 & -1 \\ & -1 & 1 & 0 & 0 \\ & 0 & 0 & -1 & 1 \\ & 1 & -1 & 0 & 0 \\ & 0 & 0 & 1 & -1 \end{array}\right]$$

$\boxed{\mathbf{3}}$ Define $[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}]$. The geometry property holds since for $\ell m \in V \cup E$,

$$\vartheta_{\ell m} = \underbrace{\psi_{\ell m}}_{\text{bicubic Hermite}} + \underbrace{\sum_{ij \in D} h_{ij}^{\ell m} \psi_{ij}}_{\text{zero on boundary}} \implies \vartheta_{\ell m} \equiv \psi_{\ell m} \text{ on edges}$$

Cubic Hermite Serendipity Geom. Decomp: 2D

Proof Details:

To prove that the approximation property holds, observe:

$$[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}] \quad \text{implies} \quad \sum_{ij} h_{ij}^{\ell m}\psi_{ij} = \vartheta_{\ell m}$$

For all (r, s) pairs such that $\text{sldeg}(x^r y^s) \leq 3$, the matrix entries in column ij satisfy

$$\varepsilon_{r,i}\varepsilon_{s,j} = \sum_{\ell m \in V \cup E} \varepsilon_{r,\ell}\varepsilon_{s,m} h_{ij}^{\ell m}$$

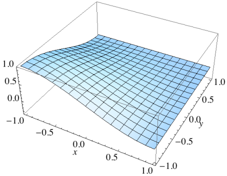
Substitute these into the Hermite 2D approximation property:

$$\begin{aligned} x^r y^s &= \sum_{ij \in V \cup E \cup D} \varepsilon_{r,i}\varepsilon_{s,j}\psi_{ij} = \sum_{ij}\sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m} h_{ij}^{\ell m}\psi_{ij} \\ &= \sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m} \sum_{ij} h_{ij}^{\ell m}\psi_{ij} = \sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m}\vartheta_{\ell m} \end{aligned}$$

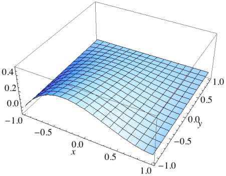
Hence $[\vartheta_{\ell m}]$ is a basis for $\mathcal{S}_2([0,1]^2)$, completing the geometric decomposition. $\square$

Hermite Style Serendipity Functions (2D)

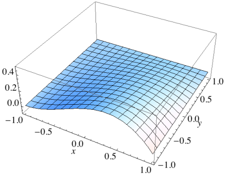
$$
[\vartheta_{\ell m}] = \begin{bmatrix} \vartheta_{11} \\ \vartheta_{14} \\ \vartheta_{41} \\ \vartheta_{44} \\ \vartheta_{12} \\ \vartheta_{13} \\ \vartheta_{42} \\ \vartheta_{43} \\ \vartheta_{21} \\ \vartheta_{31} \\ \vartheta_{24} \\ \vartheta_{34} \end{bmatrix} = \begin{bmatrix} -(x-1)(y-1)(-2+x+x^2+y+y^2) \\ (x-1)(y+1)(-2+x+x^2-y+y^2) \\ (x+1)(y-1)(-2-x+x^2+y+y^2) \\ -(x+1)(y+1)(-2-x+x^2-y+y^2) \\ -(x-1)(y-1)^2(y+1) \\ (x-1)(y-1)(y+1)^2 \\ (x+1)(y-1)^2(y+1) \\ -(x+1)(y-1)(y+1)^2 \\ -(x-1)^2(x+1)(y-1) \\ (x-1)(x+1)^2(y-1) \\ (x-1)^2(x+1)(y+1) \\ -(x-1)(x+1)^2(y+1) \end{bmatrix} \cdot \frac{1}{8}
$$



ϑ_{11}



ϑ_{21}

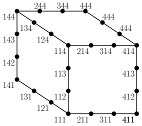


ϑ_{31}

Cubic Hermite Serendipity Geom. Decomp: 3D

Theorem [G, 2012]: A Hermite-like geometric decomposition of $\mathcal{S}_3([0,1]^3)$ exists.

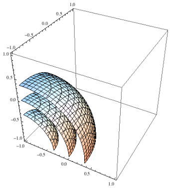
$$\underbrace{\left\{ \begin{array}{c} x^r y^s z^t \\ \text{sldeg} \leq 3 \end{array} \right\}}_{\mathcal{S}_3([0,1]^3)} \quad \longleftrightarrow \quad \left\{ \begin{array}{c} \vartheta_{\ell mn} \\ \text{(to be defined)} \end{array} \right\} \quad \longleftrightarrow$$



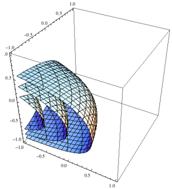
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s z^t = \sum_{\ell mn} \varepsilon_{r,i}\varepsilon_{s,j}\varepsilon_{t,k}\vartheta_{\ell mn}$, for superlinear degree$(x^r y^s z^t) \leq 3$

Geometry:



ϑ_{111}



ϑ_{112}

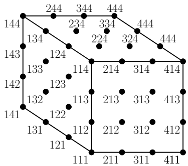
$$\begin{aligned} u &= u|_{(0,0,0)}\vartheta_{111} \\ &\quad + \partial_x u|_{(0,0,0)}\vartheta_{211} \\ &\quad + \partial_y u|_{(0,0,0)}\vartheta_{121} \\ &\quad + \partial_z u|_{(0,0,0)}\vartheta_{112} \\ &\quad + \cdots \end{aligned}$$

$\forall u \in \mathcal{S}_3([0,1]^3)$

Proof Overview:

1 Fix index sets and basis orderings based on domain points:

$$
\begin{aligned}
V &= \text{vertices } (111, \ldots) \\
E &= \text{edges } (112, \ldots) \\
F &= \text{face interior } (122, \ldots) \\
M &= \text{volume interior } (222, \ldots)
\end{aligned}
\quad \longleftrightarrow
$$



$$
[\vartheta_{\ell mn}] := [\ \vartheta_{111}, \ldots, \vartheta_{444},\ \ \vartheta_{112}, \ldots, \vartheta_{443}\ ],
$$

$$
[\psi_{ijk}] := [\ \underbrace{\psi_{111}, \ldots, \psi_{444},}_{\text{indices in } V}\ \underbrace{\psi_{112}, \ldots, \psi_{443},}_{\text{indices in } E}\ \underbrace{\psi_{122}, \ldots, \psi_{433},}_{\text{indices in } F}\ \underbrace{\psi_{222}, \ldots, \psi_{333}}_{\text{indices in } M}\ ]
$$

2 Define a 32×64 matrix $\mathbb{W}$ with entries $h_{ijk}^{\ell mn}$ (where $\ell mn \in V \cup E$)

3 Define the serendipity basis functions $\vartheta_{\ell mn}$ via

$$
\boxed{[\vartheta_{\ell mn}] := \mathbb{W}[\psi_{ijk}]}
$$

and show that the approximation and geometry properties hold.

Cubic Hermite Serendipity Geom. Decomp: 3D

Proof Details:

2 Define a 32×64 matrix $\mathbb{W}$ with entries $h_{ijk}^{\ell mn}$ so that $\ell mn \in V \cup E$

$$\mathbb{W} := \left[\begin{array}{c|c} \mathbb{I} & \text{specific full rank} \\ \text{(32x32 identity matrix)} & \text{32x32 matrix} \\ & \text{with entries -1, 0, or 1} \end{array} \right]$$

3 Define $[\vartheta_{\ell mn}] := \mathbb{W}[\psi_{ijk}]$.

$\longrightarrow$ Confirm directly that $[\vartheta_{\ell mn}]$ restricts to $[\vartheta_{\ell m}]$ on faces.

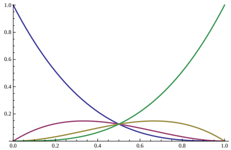
$\longrightarrow$ Similar proof technique confirms geometry and approximation properties.

$$[\vartheta_{\ell mn}] = \begin{bmatrix} \vartheta_{111} \\ \vartheta_{114} \\ \vdots \\ \vartheta_{442} \\ \vartheta_{443} \end{bmatrix} = \begin{bmatrix} (x-1)(y-1)(z-1)(-2+x+x^2+y+y^2+z+z^2) \\ -(x-1)(y-1)(z+1)(-2+x+x^2+y+y^2-z+z^2) \\ \vdots \\ (x+1)(y+1)(z-1)^2(z+1) \\ -(x+1)(y+1)(z-1)(z+1)^2 \end{bmatrix} \cdot \frac{1}{16}$$

Cubic Bernstein Serendipity Geom. Decomp: 2D, 3D

Cubic Bernstein Basis on $[0,1]$

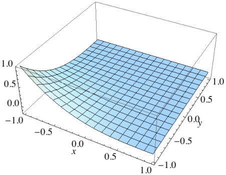
$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} := \begin{bmatrix}(1-x)^3\\ (1-x)^2x\\ (1-x)x^2\\ x^3\end{bmatrix}$$



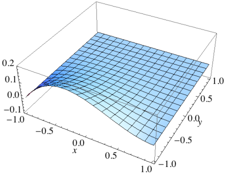
Theorem [G, 2012]: Bernstein-like geometric decompositions of $\mathcal{S}_3([0,1]^2)$ and $\mathcal{S}_3([0,1]^3)$ exist.

$$[\xi_{\ell m}] = \begin{bmatrix}\xi_{11}\\ \xi_{14}\\ \vdots\\ \xi_{24}\\ \xi_{34}\end{bmatrix} = \begin{bmatrix}(x-1)(y-1)(-2-2x+x^2-2y+y^2)\\ -(x-1)(y+1)(-2-2x+x^2+2y+y^2)\\ \vdots\\ (x-1)^2(x+1)(y+1)\\ -(x-1)(x+1)^2(y+1)\end{bmatrix} \cdot \frac{1}{16}$$

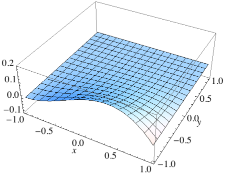
$$[\xi_{\ell mn}] = \begin{bmatrix}\xi_{111}\\ \xi_{114}\\ \vdots\\ \xi_{442}\\ \xi_{443}\end{bmatrix} = \begin{bmatrix}-(x-1)(y-1)(z-1)(-5-2x+x^2-2y+y^2-2z+z^2)\\ (x-1)(y-1)(z+1)(-5-2x+x^2-2y+y^2+2z+z^2)\\ \vdots\\ (x+1)(y+1)(z-1)^2(z+1)\\ -(x+1)(y+1)(z-1)(z+1)^2\end{bmatrix} \cdot \frac{1}{32}$$



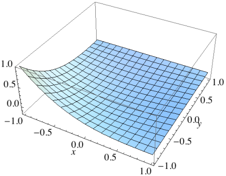
β_{11}



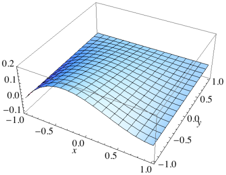
β_{21}



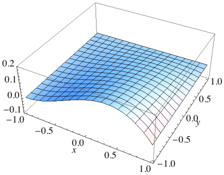
β_{31}



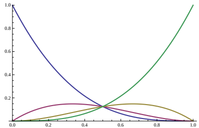
ξ_{11}



ξ_{21}



ξ_{31}



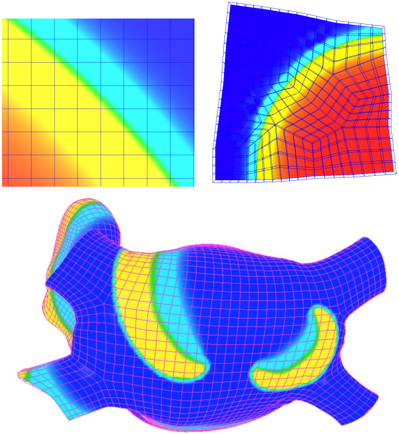
Bicubic Bernstein functions (top) and Bernstein-style serendipity functions (bottom).

$\rightarrow$ Note boundary agreement with Bernstein functions.

Outline

1. The Cubic Case: Hermite Functions, Serendipity Spaces
2. Geometric Decompsitions of Cubic Serendipity Spaces
3. Applications and Future Directions

Application: Cardiac Electrophysiology



→ Cubic Hermite serendipity functions recently incorporated into Continuity software package for cardiac electrophysiology models.

→ Used to solve the *monodomain* equations, a type of reaction-diffusion equation

→ Initial results show agreement of serendipity and standard bicubics on a benchmark problem with a $\boxed{\textbf{4x computational speedup}}$ in 3D.

→ Fast computation essential to clinical applications and 'real time' simulations

GONZALEZ, VINCENT, G., MCCULLOCH *High Order Interpolation Methods in Cardiac Electrophysiology Simulation*, in preparation, 2013.

Future Directions and Open Questions

$\rightarrow$ Applications to problems using Bernstein tensor product bases

$\rightarrow$ Analysis of the use of serendipity bases for geometric modeling

$\rightarrow$ Construction of bases for $\mathcal{S}_r\Lambda^k([0,1]^n)$ for
- $\rightarrow$ Higher order scalar cases ($k = 0$, $r > 3$, $n = 2, 3$)
- $\rightarrow$ Higher form order cases ($k > 0$)

Acknowledgments

Matt Gonzalez	UC San Diego
Kevin Vincent	UC San Diego
Andrew McCulloch	UC San Diego
Michael Holst	UC San Diego
National Biomedical Computation Resource	UC San Diego

Thanks to the NSF and conference organizers for travel support!

Slides and pre-prints: `http://ccom.ucsd.edu/~agillette`