Worksheet 2

Name:

1. The Burj Khalifa in Dubai is the world's tallest building at $2,722$ ft. A ball is dropped from the top of the building. If one ignores air resistance, then the height y of the ball above the ground (in feet) is a given as a function of time t (in seconds) by the formula

$$y(t) = 2722 - 16t^2.$$

(a) Find the velocity of the ball at time t.

(b) How long will it take for the ball hit the ground? Round your answer to the nearest 0.01 second.

(c) Find the velocity of the ball right the instant before it hits the ground. Round your answer to the nearest 0.01 $\frac{\text{ft}}{\text{sec}}$.

2. Let $P(r)$ be the quadratic function $P(r) = \frac{1}{2}r^2 - (\sqrt{3})\,r + 1$. Use differentiation to answer (with NO decimal approximations) the following:

(a) Compute $P'(r)$.

(b) On which interval(s) is $P(r)$ decreasing?

(c) Compute $P''(r)$.

(d) On which interval(s) is $P(r)$ concave up?

3. A particle's position y (in meters) is given as a function of t (in seconds). Match each expression on the left with its corresponding physical interpretation on the right.

Expression	Physical Interpretation
$y(2)$	
$\frac{y(5)-y(4)}{5-4}$	
$y'(2)$	
$y''(2)$	

List of Physical Interpretions :

I. The velocity of the particle at $t = 2$ seconds.

II. The particle's position at $t = 2$ seconds.

III. The particle's acceleration at $t = 2$ seconds.

IV. The average acceleration of the particle on the interval $[4, 5]$

V. The average velocity of the particle on the interval $[4, 5]$

4. Let $f(x) = \dfrac{1}{x}$.

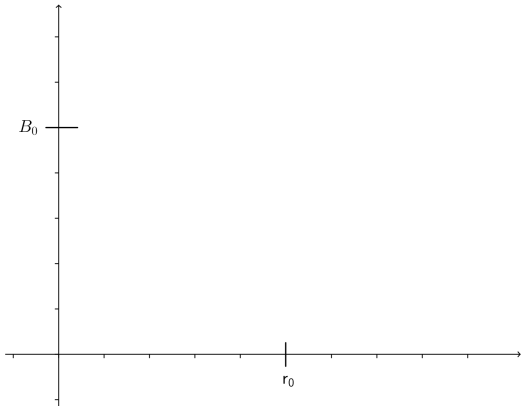
(a) Write an expression for the average rate of change of $f(x)$ on the interval $[x, x+h]$.

(b) Compute $f'(x)$ by using the definition of the derivative.

5. Let B_0 be a positive constant. A magnetic field B is given as a function of the distance, r, from the center of a wire of radius r_0 as follows:

$$B(r) = \begin{cases} \frac{r}{r_0} B_0 & 0 < r \leq r_0 \\ \frac{r_0}{r} B_0 & r > r_0 \end{cases}$$

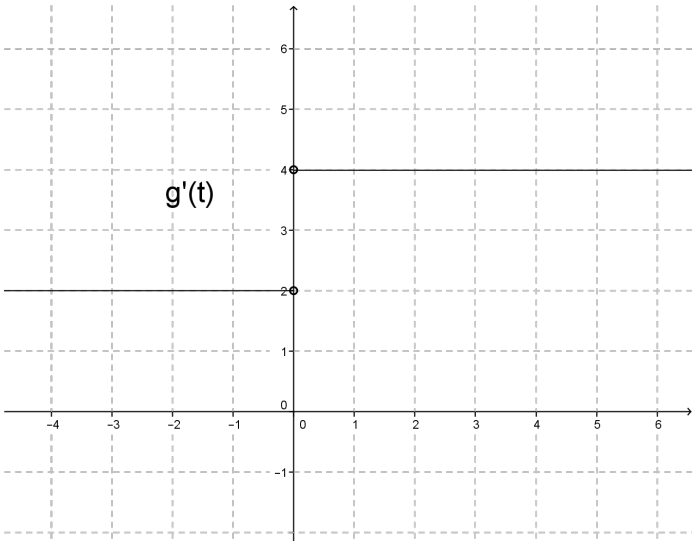
(a) Sketch a qualitative graph of $y = B(r)$ below.



(b) Is B continuous at $r = r_0$? Answer Yes or No and provide an explanation.

(c) Is B differentiable at $r = r_0$? Answer Yes or No and provide an explanation.

6. The graph of $y = g'(t)$ is shown below.



Sketch the graph of $y = g(t)$ below. Assume $g(0) = 0$.