Homework 6
(due Friday, November 7)

1. Consider the family of functions $f(\theta) = a\cos(b\theta^2)$ for $\theta > 0$. Find the values of a and b so that the following properties hold:

(i) The first critical point of $f(\theta)$ occurs at $\theta = 1$

(ii) $f'(\frac{1}{\sqrt{2}}) = -2$

2. Suppose A and B are positive constants. The force between two atoms is given by $F(r) = -\frac{A}{r^2} + \frac{B}{r^3}$, where $r > 0$ is the distance between them.

(a) Find the horizontal and vertical intercepts

(b) Find the vertical and horizontal aymptotes

(c) Find the critical points of $F(r)$

(d) Classifiy the critical points (are they local maxima, local minima or neither?)

(e) Find the inflection points of $F(r)$

3. Suppose a farmer wants to enclose a rectangular garden but he only has 100 feet of fencing. What is the maximum area that he can enclose?

4. A rectangle has one side on the x-axis, one side on the y-axis, one corner at the origin and one corner on the curve $y = e^{-2x}$ for $x > 0$.

(a) Find the dimensions of the rectangle with maximum area

(b) Find the dimensions of the rectangle with minimum perimeter

5. A company manufactures cylindrical barrels to store nuclear waste. The top and bottom of the barrels are to be made with material that costs \$10 per square-foot and the rest is to be made of material that costs \$8 per square-foot. If each barrel is to hold 5 cubic feet, then find the dimensions of the barrel that will minimize the cost.