Homework 12
(due Wednesday, December 10)

1. Use the formula $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ to represent $g'(0)$ for the function $g(x) = \tan(x)$.

2. Suppose that the number of cars of a certain model sold (represented by the letter N) is a function of its price p. Suppose p is given in thousands of dollars.

(a) Give a practical interpretation of $N'(12) = -10$.

(b) If $N'(12) = -10$ and $N(12) = 21$, then find the equation of the tangent line to $N(p)$ at $p = 12$.

(c) Use your answer in part (b) to estimate $N(13)$.

(d) If $N''(p) > 0$ for all p, then is your esimate in part (c) an overestimate or an underestimate?

3. Find the derivative of each function.

(a) $y = \ln(\sin(x)) + x^e + e^x$

(b) $f(z) = \dfrac{Az^2 + B}{Cz^2 + D}$, where A, B, C and D are non-zero constants.

(c) $g(t) = \dfrac{1}{\cos(2t)} + \arcsin(t^2 - 1)$

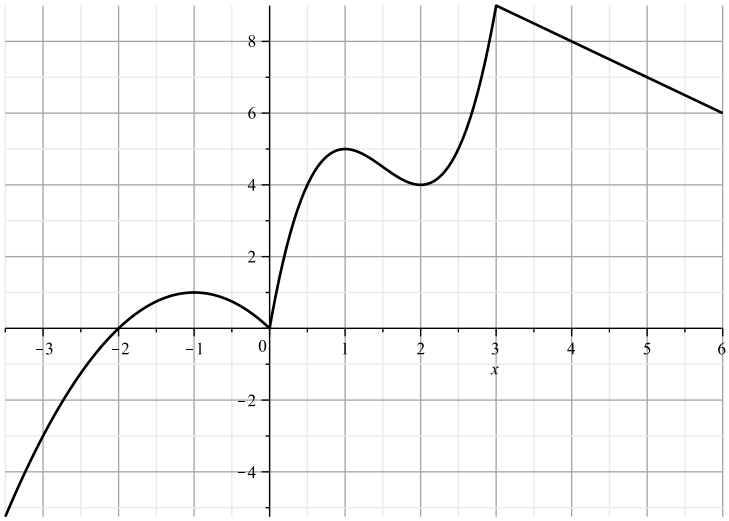
4. Suppose k is a positive constant. Find the critical points of the function

$$S = \frac{7k}{x^2} + \frac{k}{(20 - x)^2},$$

where $0 < x < 20$. Use the first derivative test to determine if the critical points correspond to local maxima or local minima.

5. Compute the limit $\lim_{x \to 0} \dfrac{xe^{2x}}{3x + \sin(2x)}$. If L'hopital's rule applies, then you must use it.

6. Sketch a possible graph of $f'(x)$ given the graph of $f(x)$ shown below.



7. Consider $x^2 + y^2 - 4x + 7y = 15$.

(a) Find $\dfrac{dy}{dx}$

(b) Under what conditions on x and/or y is the tangent line to this curve horizontal? Vertical?

8. Find the following anti-derivatives.

(a) $\displaystyle\int \left(Ax^2 + \frac{B}{1+x^2}\right) dx$

(b) $\displaystyle\int \frac{y + 2y^2}{y^3}\, dy$

(c) $\displaystyle\int \sqrt[3]{t}(t^2 + At^3)\, dt$

9. (Integration by Substitution) Compute

(a) $\displaystyle\int \frac{x}{1+3x^2}\,dx$

(b) $\displaystyle\int t\cos(t^2)\,dt$

10. Suppose $\displaystyle\int_2^7 f(x)\,dx = -2$, $\displaystyle\int_2^7 g(x)\,dx = 4$ and $\displaystyle\int_0^{10} h(x)\,dx = 12$.

(a) Find $\displaystyle\int_2^6 \big(3f(x) - g(x)\big)\,dx$

(b) Find $\displaystyle\int_0^{10} \big(h(x) + A\big)\,dx$, where A is a non-zero constant.

(c) If $h(x)$ is an odd function, then find $\displaystyle\int_{-10}^{10} h(x)\,dx$.

(d) Suppose $f(x) = F'(x)$ and $F(2) = 10$. Find $F(7)$.

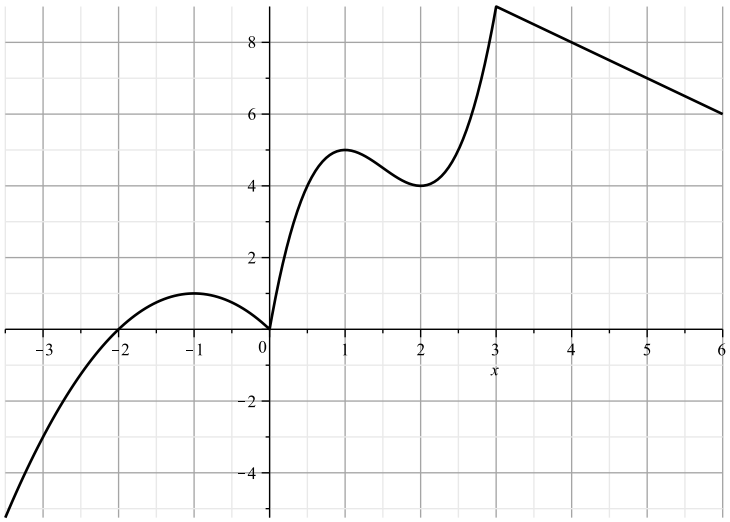
11. Do problem 38 from section 5.3.

12. The volume of a pyramid with a square base is given by $V = \frac{1}{3}x^2y$, where x is the length of the square's sides and y is the hight of the pyramid.

(a) How does the pyramid's volume change with respect to its height?

(b) Suppose the height of the pyramid is decreasing at a rate of 2 inches per year and the length of the base's sides are increasing at a rate of 5 inches per year. Find the rate of change of the pyramid's volume when $x = 100$ and $y = 200$.

13. Suppose the graph of $y = g'(x)$ is shown below.



(a) Find the critical points of $g(x)$.

(b) Compute $\displaystyle\int_{-3}^{6} g''(x)\,dx$

(c) Is $\displaystyle\int_{-3}^{2} g'(x)\,dx$ positive or negative?

(d) Is $g(x)$ increasing or decreasing on the interval $-1 < x < 0$?

14. A fence is 8 feet tall runs parallel to a tall building at a distance of 4 feet from the building. Find the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.