

Homework 11
(due Monday, December 8)

1. Print out the final exam study guide and the Useful Formulas sheet. There are links on the course website under “Resources for Students.”

2. Use the formula $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to represent $f'(5)$ for the function $f(x) = 2^x$.

3. Suppose that the population P (in millions of people) of the United States is a function of the year t , where t is given in years after 1900.

- (a) Give a practical interpretation of $P'(100) = 2$.
- (b) If $P'(100) = 2$ and $P(100) = 300$, then find the equation of the tangent line to $P(t)$ at $t = 100$.
- (c) Use your answer in part (b) to estimate $P(103)$.
- (d) If $P''(t) < 0$ for all t , then is your estimate in part (c) an overestimate or an underestimate?

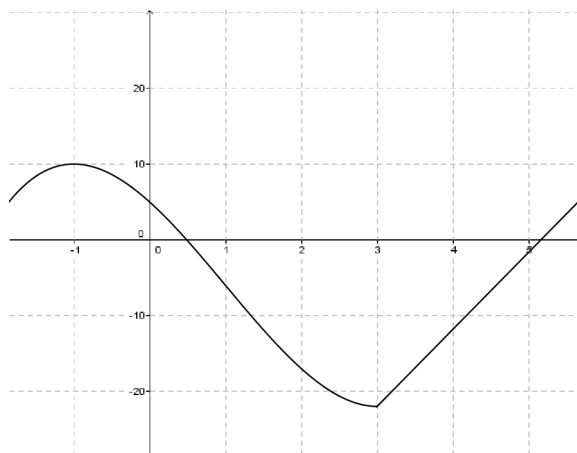
4. Find the derivative of each function.

- (a) $y = \ln(1 - x^2) + \ln(\pi)$
- (b) $f(z) = \frac{Az + B}{Cz + D} + \frac{e}{z^2}$, where A, B, C and D are non-zero constants.
- (c) $g(t) = \frac{1}{\sin(2t)} + \arctan(t^2 - 1)$

5. Consider the family of functions defined by $F(x) = ax^2 \ln(x) + kx$, where a and k are non-zero constants. Find the values of a and k so that $f(x)$ has a critical point at $(1, 10)$.

6. Compute the limit $\lim_{t \rightarrow 0} \frac{t2^t}{2t + \sin(t)}$. If L’hopital’s rule applies, then you must use it.

7. Sketch a possible graph of $f'(x)$ given the graph of $f(x)$ shown below.



8. Consider $2y^5 + 6x^2 - 4 = 7x + 5y$.

(a) Find $\frac{dy}{dx}$

(b) Find the y -coordinate(s) where the tangent line is vertical.

9. Find the following anti-derivatives.

(a) $\int (Kx + 4 \sec^2(x)) dx$

(b) $\int \frac{y+2}{\sqrt{y}} dy$

(c) $\int \sqrt{t}(t^2 + t) dt$

10. (Integration by Substitution)

(a) Compute $\int y^2 \sqrt{8 - 4y^3} dy$

(b) Compute $\int_0^1 \frac{3^x}{3^x + 1} dx$