BER of a Truncated Viterbi Dector

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***Abstract*—We perform a simulated study to evaluate the bit-error rate (BER) of a Viterbi detector.**

I. Introduction

Fix a positive integer $1 \leq M \leq 256$ and a binary sequence of inputs $m = (m_i)_{\imath \geq 1}$. Suppose we have a finite-state machine (FSM) with state values in $S = \{1, \ldots, M-1\}$ and output values in some finite set X with q elements. Given the state $s_i \in S$ and input m_i we set $s_{i+1} = N(s_i, m_i)$, and we let $x_i = O(s_i, m_i) \in X$ denote the output of the FSM. Here, O and N are given as a $M \times 2$-arrays. We will denote our sequence of outputs $x = (x_i)_{i \geq 1}$. The sequence x is passed through the AWGN channel, producing the sequence $y = (y_i)_{i \geq 1}$. The sequence y is then fed to our Viterbi detector which produces the sequence $\hat{m} = (\hat{m}_i)_{i \geq 1}$. The block diagram of our system is show below:

$$\xrightarrow{m} \boxed{\text{FSM}} \xrightarrow{x} \boxed{\text{AWGN Ch.}} \xrightarrow{y} \boxed{\text{Viterbi Detector}} \xrightarrow{\hat{m}}$$

We are interested in computing the BER of our Viterbi detector.

II. AWGN Channel

We use a Box-Muller transform to generate our AWGN channel. The Box-Muller method is straightforward. If U_1 and U_2 are independent random variables uniformly distributed in $[0, 1]$, then

$$Z_0 := \sqrt{-2 \ln U_1} \cos(2\pi U_2) \quad \text{and} \quad Z_1 := \sqrt{-2 \ln U_1} \sin(2\pi U_2)$$

are indpendent with a standard normal distribution. (see [2])

III. Detector

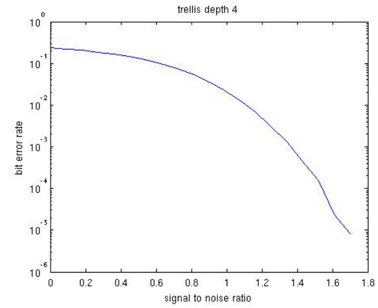
Material in this section can be found in [1]. In this project we use a truncated version of Viterbi's algorithm: Fix $n >> 0$. (In our study we take $n = 10^6$. Instead of scanning over all paths among states $0 \leq i \leq n$ we make a decision at stage $i = kD$ ($k = 1, 2, \ldots$), *i.e.,* an optimal path (among all paths prior to stage $kD - D$) is chosen at stage $i = kD$. Here, $D \leq n$ denotes a *truncation depth*. Note that as D approaches n, this truncated algorithm becomes more optimal, but at the cost of increased delay. Of course taking $D = n$ gives the original Viterbi algorithm.

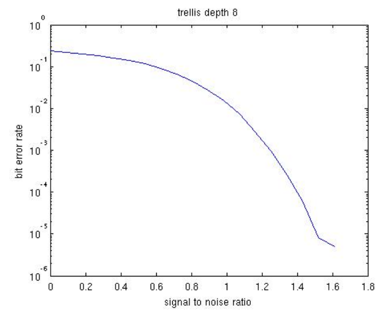
IV. Experiment

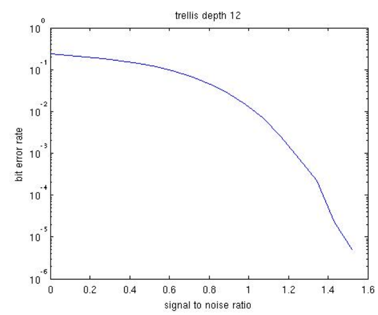
For our analysis we take $M = 4$ and $q = 5$. Let $X = \{-2, -1, 0, 1, 2\}$, and suppose the output and state-transitions are given respectively as

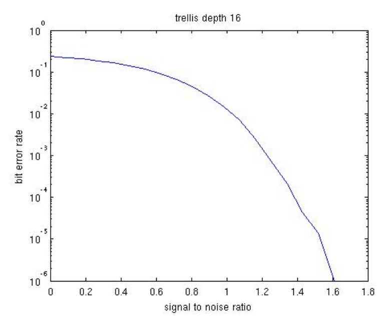
$$N = \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 0 & 1 \\ 2 & 3 \end{pmatrix} \quad \text{and} \quad O = \begin{pmatrix} -2 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 2 \end{pmatrix}.$$

Our experiment is repeated for a trellis of depth $D = 4, 8, 12, 16$. Our bit stream has length 10^6. Below are plots of BER against SNR.









References

[1] *Veterbi Algorithm*, Notes Distributed for ECE-535, http://www2.engr.arizona.edu/ṽasicteach/teaching/ece535/notes/notes_050_viterbi_algorithm.pdf

[2] *Box-Muller Tansform*, http://en.wikipedia.org/wiki/Box-Muller_transform