

# Graphs and their Applications

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- 4 Review

# Objectives

After today we will be able to

- (1) Describe what a graph is and describe some properties
- (2) Use graphs to answer two interesting types of questions:  
counting and sorting

# What is a Graph?

In this talk a *graph* will mean:

- (1) A collection of nodes (called *vertices*), where
- (2) some pairs may be connected by a link (called an edge)

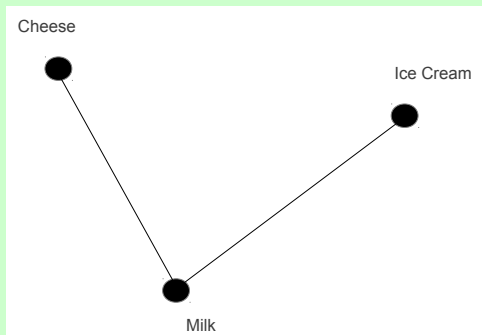
# What is a Graph?

**Warmup Exercise 1.** Come up with your own graph. Make sure it has at least 5 vertices and 4 edges.

# What is a Graph?

Vertices can be labeled and so graphs can be a useful way to encode information.

**Example.** What might the following graph represent?



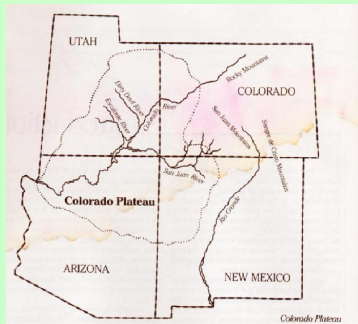
# What is a Graph?

Now let's come up with a graph right now that encodes information about us...

Each vertex will represent a person in this room, and vertex A will be connected to vertex B if person A and person B had met before today.

# Coloring

**Question.** What is the minimum number of colors we need to color the map so that no adjacent states share the same color?



(image source [http://www.drabruzzo.com/culture\\_areas.htm](http://www.drabruzzo.com/culture_areas.htm))

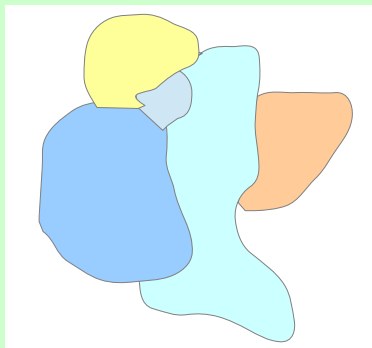
# Coloring

**Warmup Exercise 2.** Come up with a map of 4 states that requires *three* colors in order to be colored in such a way that no adjacent states share the same color.

# Coloring

Now let's try a different question...

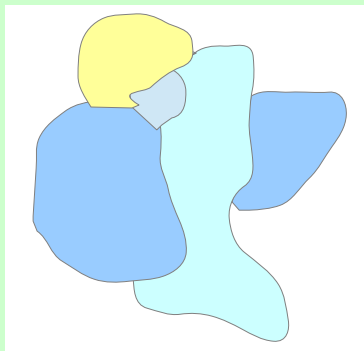
**Question.** Can the following map be colored with fewer than five colors? Can it be colored with three colors?



# Coloring

Now let's try a different question...

**Question.** Can the following map be colored with fewer than five colors? Can it be colored with three colors?



# Coloring

In general, only four colors are needed to color *any* map. This is known as the **Four-Color Theorem**.

Students can try to color various maps with as few colors as possible so that no adjacent states (or counties) share the same color.

Students can arrive to the conclusion of the four color theorem on their own!

# Planarity

**Question.** What do maps have to do with graphs?

**Answer.** Maps have graph representations and (some graphs) have map representations.

**Warmup Exercise 3.** Construct a graph representation of your four-state map from Warmup Exercise 2

# Planarity

It turns out that graphs that correspond to colored maps will always be *planar graphs* (they can be drawn with no intersecting edges)

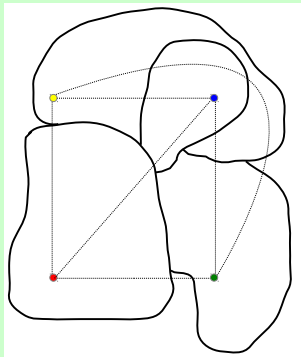
**Warmup Exercise 4.** Can you draw a graph with 4 vertices so each vertex is connected to each other vertex and with the property that no edges intersect? How about 5 vertices?

# Planarity

**Warmup Exercise 5.** Draw a map that corresponds to your graphs from Warmup Exercise 4.

# Planarity

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# Planarity and Coloring

**Question.** What might we suspect about planar graphs in terms of coloring?

# Planarity and Coloring

**Question.** What might we suspect about planar graphs in terms of coloring?

**Fact.** The vertices of any planar graph can be colored (using at most 4 colors) so that no adjacent vertex shares the same color !

# A Sorting Problem

Suppose you are an animal enthusiast and you want to build terrariums for your different critters. However, you must be careful since some of your animals are natural predators of some of your other animals.

In **Activity 1** we will find the smallest number of terrariums that we need to construct

# Counting Edges

The graphs we will be working with in **Activity 2** are called the *complete graphs*

These are graphs where there is a link between any pair of vertices.

**Question.** Which Warmup Exercise asked you to draw the complete graphs on 4 and 5 vertices?

# Counting Edges

**Question.** How many edges does the complete graph on  $n$  vertices have?

Let's try to answer this question in the context of combinations in **Activity 2...**

## What did we Learn?

1. Graphs consist of vertices and edges
2. Graphs can use useful tools to encode information
3. Graphs whose edges don't intersect are called planar graphs
4. Any map can be represented by a planar graph
5. The vertices in a planar graph can be colored (with  $\leq 4$  colors) in such a way so that no adjacent vertices share the same color
6. The complete graph on  $n$  vertices has  $\frac{n(n-1)}{2}$  edges, which represents the number of ways to choose two objects from a list of  $n$  objects

# References

- [1] <http://www.colorado.edu/education/DMP>
- [2] [http://www.drabruzzo.com/culture\\_areas.htm](http://www.drabruzzo.com/culture_areas.htm)