

Homework 14: Differential Equations (due April 24)

1. A quantity Q satisfies the differential equation

$$\frac{dQ}{dt} = \frac{t}{Q} - 0.5$$

If $Q = 8$ when $t = 2$, use $\frac{dQ}{dt}$ to determine whether Q is increasing or decreasing at $t = 2$.

2. Show that $y = A + Ce^{kt}$ is a solution of the differential equation

$$\frac{dy}{dt} = k(y - A)$$

3. The height y of a hanging cable satisfies the differential equation

$$\frac{d^2y}{dx^2} = k\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

(a) Show that $y = \frac{e^x + e^{-x}}{2}$ satisfies the differential equation if $k = 1$.

(b) For general k , one solution to this differential equation is of the form

$$y = \frac{e^{Ax} + e^{-Ax}}{2A}.$$

Substitute this expression for y into the differential equation to find A in terms of k .