Homework 13: Taylor Polynomials and Taylor Series Part II (due April 20)

1. Find the Taylor polynomial approximation of $\sqrt{1-x}$ near $a = 0$ with degree $n = 3$.

2. Find the Taylor polynomial approximation of $\ln(1+x)$ near $a = 0$ with degree $n = 4$.

3. Find the Taylor polynomial approximation of $\arctan(x)$ near $a = 1$ with degree $n = 3$.

4. Find the radius of convergence of the Taylor series around $x = 0$ for $\ln(1-x)$.

5. Evaluate the following convergent series by recognizing it as a Taylor series evaluated at a particular value of x:

$$1 - \frac{100}{2!} + \frac{10000}{4!} + \cdots + \frac{(-1)^n\, 10^{2n}}{(2n)!} + \cdots$$

6. Evaluate the following convergent series by recognizing it as a Taylor series evaluated at a particular value of x:

$$1 + 3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$$

7. Use your knowledge about the Taylor series of $\sin(x)$ about $x = 0$ to find the Taylor series of $\sin(2x^3)$ about $x = 0$.

8. Find the Taylor series about 0 for the function $f(x) = x\ln(1+2x)$. Include the general term.