

Homework 12: Taylor Polynomials and Taylor Series Part I (due April 17)

1. Show that the Taylor series of $f(x) = e^x$ about $x = 0$ is given by

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

2. Show that the Taylor series of $f(x) = \sin(x)$ about $x = 0$ is given by

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

3. Show that the Taylor series of $f(x) = \cos(x)$ about $x = 0$ is given by

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

4. Show that the interval of convergence for the Taylor series expansion for e^x is actually $(-\infty, \infty)$ (The same is true for the expansions of $\sin(x)$ and $\cos(x)$ but you don't need to show this).